# Yielding of Brass Case Walls in the Chamber

## A Technical Note

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## 1.0 Introduction

This note is to document the technical details behind my proposed ***Precision Shooting*** article entitled *Steel Support for the Brass Cartridge Case*. In that article, I continue the analysis of an example target rifle chambered in .308 Winchester and made on a blueprinted Remington 700 bolt action. I am greatly indebted to Al Harral of Livermore Software Technology Corporation (LSTC) for his gentle corrections and for running several Finite Element Analysis (FEA) studies supporting this work using his company's sophisticated LS-DYNA program. His FEA results can be viewed online at www.VarmintAl.com/amod7.htm . This FEA study is based on a perfectly manufactured Remington Model 7 bolt action chambered in .243 Winchester, but the materials data and chamber pressure curve are identical to my example. LS-DYNA provides a "transient dynamic" computation of all inertial and vibrational effects in its mathematical models. Before any attempts are made to generalize about what happens to the brass cartridge case in firing, we should conduct several additional FEA studies for varying conditions and interpret the results carefully. The color fringe image shown in Figure 1 illustrates the effective von Mises tensile stress levels at peak chamber pressure for each separate material element of the barreled action and its chambered cartridge case. This image was computed and output by LS-DYNA and serves to demonstrate the great power of this program.

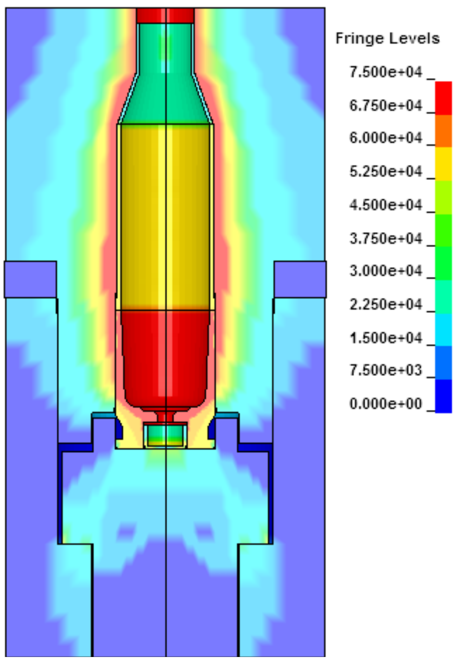


**Figure 1. Peak Effective Stress Levels**

## 1.1 Dynamic versus Quasi-Static Analyses

Even at gunpowder burning rates, *radial and tangential chamber expansions* can be calculated satisfactorily using quasi-static analysis in which pressure effects are treated just as in static equilibrium. In fact, this very situation is cited as the exemplar of "quick static loading" in ***Roark's Formulas for Stress and Strain, Seventh Edition***, Young and Budynas, McGraw Hill, 2002, page 36. The speeds, displacements and masses involved are not large enough to interfere with obtaining reasonably accurate results from this

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simpler approach. In the *axial direction*, though, the bolt-face setback ***must*** be analyzed dynamically, at least during any intervals including the sharp peak in chamber pressure. However, the *chamber pressure* itself can be treated satisfactorily as a quasi-static pressure within the chamber (and in the primer pocket, as well) up until the accelerating bullet has moved several inches down the bore where viscous gas-flow considerations become significant. This means that, at any particular instant in time, the pressure in the chamber can be treated as a hydrostatic pressure, having the same effect everywhere it reaches and equally in all directions. LS-DYNA provides *fully dynamic* modeling when it is driven by a suitable pressure-versus-time curve as shown in Figure 2.

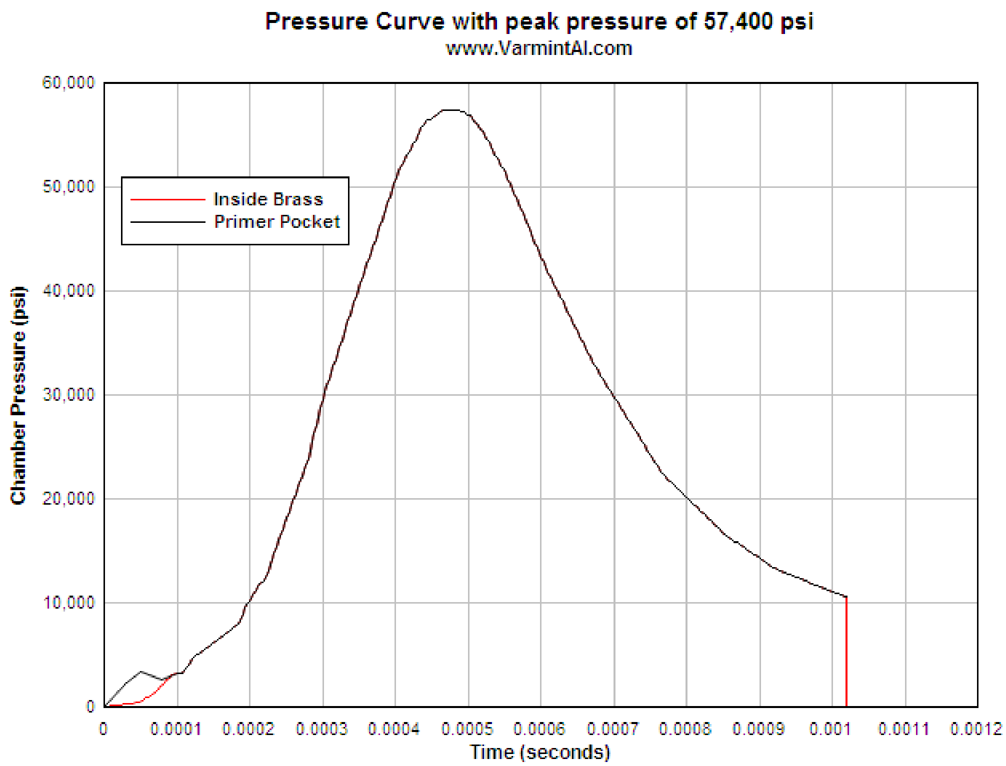


**Figure 2. Function P(t) Used in Al Harral's LS-DYNA Study**

## 2.0 Case Neck Expansion

As the precision reloaded cartridge sits in the rifle chamber ready for firing, the annealed brass of the cartridge case neck contains a certain amount of “built-in” elastic tangential (hoop) stress employed to hold the bullet securely in place. The amount of this elastic neck tension depends primarily upon how much the neck walls were expanded in internal diameter as the bullet was seated in the neck of the cartridge case. Let us calculate the chamber pressure required to release our example **168-grain** Sierra MatchKing .308 Winchester bullet being held with a typical amount of *elastic neck tension* due to having expanded the case neck by almost **2 mils** during bullet seating. The cartridge case neck

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walls are **0.014 inches** (or **14 mils**) thick in this example and are made of *annealed cartridge brass*, and are, of course, **0.308 inches** in inside diameter (ID).

The "yield point" stress level for annealed cartridge brass is **8,534 psi**, or about **45 percent** of the **19,000 psi** (or **19 ksi**) specified for the *0.5-percent offset yield strength* for this material. Stress levels up to **8.5 ksi** produce *very little or no* permanent distortion in the annealed brass. Table 1 shows a summary of the mechanical characteristics of **70Cu/30Zn** cartridge brass. This "yield point" is somewhat similar to an "elastic limit" for annealed cartridge brass, but this really soft brass is easily distorted, *both elastically and plastically, or "elasto-plastically*." Figure 3 shows a sketch of the stress and strain relations for a purely "strain hardening" material like our annealed cartridge brass.

### Table 1. Mechanical Characteristics of Cartridge Brass (70Cu/30Zn)

| Brass Hardness State | Ultimate Stress | 0.50-Percent Yield Stress | Estimated Yield Point | Yield Point Elastic Strain | Maximum Elongation |
|---|---|---|---|---|---|
| Dead Soft | 44 ksi | 11 ksi | 4.9 ksi | 0.031 % | 68 % |
| Annealed | 51 ksi | 19 ksi | 8534 psi* | 0.0533 %* | 55 % |
| Quarter Hard | 54 ksi | 40 ksi | 36.4 ksi | 0.228 % | 43 % |
| Half Hard | 62 ksi | 52 ksi | 46,960 psi* | 0.294 %* | 23 % |
| Full Hard (H04) | 76 ksi | 63 ksi | 57,110 psi* | 0.357 %* | 8.0 % |
| Extra Hard (H06) | 86 ksi | 65 ksi | 59.2 ksi | 0.370 % | 5.0 % |

*Iteratively fitted yield point values supplied by Al Harral. Other yield points are my estimates.

**<u>Common Data</u>**:

| | |
|---|---|
| Density | = 0.308 pounds/cubic inch |
| Modulus of Elasticity | = 16 msi |
| Shear Modulus | = 6.015 msi |
| Bulk Modulus | = 15.69 msi |
| Poisson's Ratio | = 0.33 |
| Coefficient of Thermal Exp. | = 11.10 per million per degree F. |

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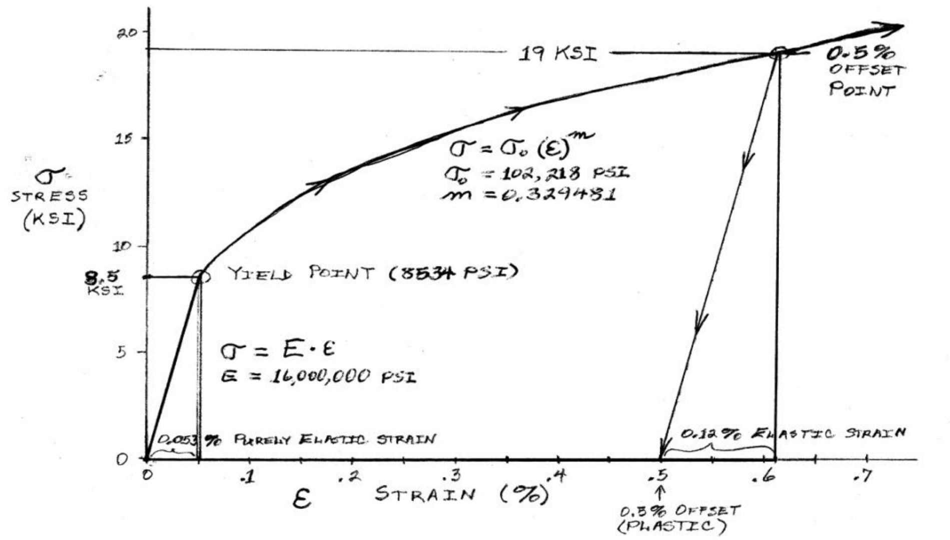


**Figure 3. Stress versus Strain Curve for "Purely Strain Hardening" Materials.**

Below this rather low "yield point" stress level of **8534 psi**, annealed brass is said to be ***purely elastic*** and to behave in accordance with Hooke's Law:

$$\sigma = E\ \varepsilon$$

where $\sigma$ = Stress (in psi),

$\varepsilon$ = Strain (a dimensionless ratio), and

**E** = Young's Modulus of Elasticity for this brass = **16 msi**.

*Above* this "yield point" stress level, annealed brass becomes ***both plastic and elastic***, or "***elasto-plastic***," and behaves according to the "strain hardening" equation:

$$\sigma = \sigma_0\ \varepsilon^m$$

where $\sigma_0$ **= 102,218 psi**, and

**m = 0.329481.**

The values, $\sigma_0$, and **m**, were iteratively fitted to the tensile test data for annealed cartridge brass by Al Harral for use in his structural analyses as shown in Table 2. Strain Hardening Data.

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### Table 2. Strain Hardening Data for Cartridge Brass

| Brass Hardness State | Yield point Stress | 0.50-Percent Offset Stress | 0.5 % Offset Total Strain | $\sigma_0$ (Stress Coefficient) | **m** (Strain Exponent) |
|---|---|---|---|---|---|
| Dead Soft | 4.9 ksi | 11 ksi | 0.569 % | 46,004 psi | 0.276784 |
| Annealed | 8534 psi* | 19,146 psi | 0.620 % | 102,218 psi* | 0.329481* |
| Quarter Hard | 36.4 ksi | 40 ksi | 0.750 % | 58,892 psi | 0.079059 |
| Half Hard | 46,960 psi* | 52 ksi | 0.825 % | 87,646 psi* | 0.10701* |
| Full Hard (H04) | 57,110 psi* | 63 ksi | 0.894 % | 110,544 psi* | 0.117193* |
| Extra Hard (H06) | 59.2 ksi | 65 ksi | 0.906 % | 106,181 psi | 0.104336 |

*Iteratively fitted yield point values supplied by Al Harral. Other data are my estimates.

Note that, *at the transitional "yield point,"*

$$\boldsymbol{\sigma_y = 8534\ psi} \text{ and}$$

$$\boldsymbol{\varepsilon_y = 0.0005334},$$

***both*** the *purely elastic* and *elasto-plastic strain* relationships are essentially satisfied:

$$\boldsymbol{\sigma_y = E\ \varepsilon_y = \sigma_0\ (\varepsilon_y)^m = 8534\ psi.}$$

As another check on this fitted strain hardening curve, consider the *0.5 percent offset yield strength* rating of **19 ksi** for annealed cartridge brass. If we use $\boldsymbol{\sigma_{off} = 19{,}146\ psi}$ for this strength rating, the *total strain* at this stress level would be:

$$\boldsymbol{\varepsilon_{off} = 0.005000} \text{ (offset)} \boldsymbol{+ [(19{,}146\ psi) / E] = 0.006197} \text{ and}$$

$$\boldsymbol{\sigma_{off} = (102{,}218\ psi)*(0.006197)\text{^}(0.329481) = 19{,}146\ psi.\ \ [QED]}$$

We will work through a single example here in which the case neck tension holding the seated bullet happens to be the standard *0.5 percent offset yield strength rating* of **19,146 psi** for our annealed cartridge brass. This is also a reasonable neck tension value that would result from expanding our example case neck by just under **2 mils** during bullet seating. This **19 ksi** *0.5-percent yield stress* first occurs as chamber pressure reaches **1800 psi**. These neck walls will expand to the full **3.0 percent** strain required for contact with the **0.3462-inch** inside diameter of the neck of our minimum-SAAMI-spec match chamber with only **1300 psi** of additional chamber pressure. The maximum elongation for annealed cartridge brass is specified to be **55 percent** before rupturing or fracturing, so there should be no danger of failure with brass cases in good condition.

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Questions sometimes arise as to how much ***bullet seating force*** would be required if the annealed case neck walls had to be ***radially expanded*** in inside diameter by differing amounts during the seating operation. Table 3 shows this variation in required seating force for our example bullet when the *coefficient of sliding friction* is assumed to be **0.30**. The first row in the table corresponds to the *maximum purely elastic expansion* of **0.1645 mils**. The tabulated neck expansions up to perhaps **3 mils** could be reasonable for our .308 Winchester match loads. The neck expansions of **5, 10, and 20 mils** were included just to show how the required seating force would continue to increase at ever-reducing rates.

**Table 3. Seating Force vs. Neck Expansion for Annealed Brass**

| Case Neck Expansion (mils) | Total Strain (%)[1] | Stress (psi)[2] | Elastic Strain %[3] | Plastic Strain %[4] | Pressure (psi)[5] | Seating Force (lbs)[6] |
|---|---|---|---|---|---|---|
| 0.1645 | 0.0534 | 8538 | 0.0534 | 0.0000 | 820.6 | 73.8 |
| **0.5** | **0.1623** | **12314** | **0.0770** | **0.0854** | **1183.6** | **106.5** |
| **1.0** | **0.3247** | **15474** | **0.0967** | **0.2280** | **1487.2** | **133.8** |
| **1.5** | **0.4870** | **17685** | **0.1105** | **0.3765** | **1699.8** | **153.0** |
| **1.9085** | **0.6196** | **19146** | **0.1197** | **0.5000** | **1840.2** | **165.6** |
| **2.0** | **0.6494** | **19444** | **0.1215** | **0.5278** | **1868.8** | **168.2** |
| **3.0** | **0.9740** | **22223** | **0.1389** | **0.8351** | **2135.9** | **192.2** |
| 5.0 | 1.6234 | 26296 | 0.1644 | 1.4590 | 2527.4 | 227.4 |
| 10.0 | 3.2468 | 33043 | 0.2065 | 3.0402 | 3175.9 | 285.8 |
| 20.0 | 6.4935 | 41520 | 0.2595 | 6.2340 | 3990.7 | 359.1 |

| | | |
|---|---|---|
| **Neck ID (2r)** | **0.308 in.** | **[1]Strain = Neck Expansion / Neck ID** |
| **Wall Th.= dr** | **0.014 in.** | **[2]Stress = (102,218 psi)*(Strain)^0.329481** |
| **Ratio k=r/2dr** | **5.5** | **[This is Al Harral's Fitted Curve for Strain Hardening Equation]** |
| **Pi** | **3.1415927** | **[3]Elastic Strain = Stress / Modulus Of Elasticity (E)** |
| **Contact Length** | **0.31 in.** | **[4]Plastic Strain = Total Strain - Elastic Strain** |
| **Cf** | **0.3** | **[5]Pressure = Stress/[1+3k+3k^2]^0.5** |
| **Elasticity E (psi)** | **16000000** | **[6]Seating Force = Cf*(Pi*ID*Length)*(Pressure)** |

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This table is freely available as an active MS Excel 1997 spreadsheet application that can be used to calculate required bullet seating forces for any desired caliber, neck wall thickness, contact length, or friction coefficient. Other strain hardening coefficients could easily be edited in if they are known for a different hardness state of the brass case neck material.

When compared with actual measurements, these calculated bullet seating forces seem to be running *too large by a factor of about two to three*. I can identify two of the possible geometric, mechanical effects that are ignored in this simple "radial expansion" analysis, but that would cause us to calculate *significantly smaller bullet seating forces* if they could be included. The largest effect being ignored could be termed the "transition cone" effect. As the bullet is being seated into the case neck, it pushes a traveling "pseudo-cone" of neck wall material ahead of its base. The cone angle in the vicinity of the leading edge of the base of the bullet would increase *disproportionately* as larger amounts of neck expansion are attempted. After being swaged up, the brass material in this area tends to "overshoot" the actual bullet diameter because it needs to re-bend into a cylindrical shape once again. The tensile stress stored in the case neck is ***reduced*** by this effect throughout the "expanded cylinder" portion of the enlarged neck when compared to the *simple radial expansion* calculated in the spreadsheet. In fact, this *reduction in neck tension* and in bullet seating force *builds up so significantly* with increasing neck expansions that it *becomes dominant* and accounts for the ***relative maximum in neck tension*** that we find at about 1.5 to 2.0 mils of neck expansion. The other significant effect being ignored can be called the "Chinese finger-trap" effect. This "friction multiplier" effect comes about whenever we attempt to slide a thin-walled sleeve upon a mandrel having a slight interference fit inside that sleeve. If we *push* the sleeve from the rear (as we are doing here in bullet seating), both the normal force of the sleeve upon the mandrel and its resulting internal friction force are *significantly reduced*. [An axially compressed, short, thin-walled cylinder will tend to assume the familiar "barrel shape" as it enlarges in diameter with each of its two ends constrained from freely expanding.] However, if we were to *pull* the sleeve along by its front edge (as would happen during "bullet pulling"), the friction force on the mandrel inside the sleeve would be *greatly increased* by the converse of this same effect. Hence, we have the familiar "bamboo," or "Chinese," "finger-trap" mechanism that we utilize in woven wire to enhance the "grip" of cable-end pulling devices.

## 2.1 Calculating Theoretical Bullet Seating Force

Let us consider the ring of annealed brass at the rearmost end of the case neck, just ahead of the case shoulders. For conceptual simplicity, let us stipulate that the rear portion of the full-diameter body of the seated, boat-tailed bullet does not quite internally cover this ring. At this location at the back end of the case neck:

Case neck ID **= 2r = 0.308 inch**

Neck wall thickness **= dr = 0.014 inch**

Let us define an intermediate value **k** to be the local value of the ratio **r/2dr**:

**k = 2r/4dr = 0.308/0.056 = 5.500** (here in the case neck).

Let the "transducer" chamber pressure **P** at any time **t**, as shown in Figure 2, be denoted by the function **P(t)**.

Then, the three mutually perpendicular ***principal stresses*** acting on each part of this ring element can be expressed symbolically as:

***Radial*** Stress **Sr = -P(t)** (directed inwardly),

***Tangential*** (Hoop) Stress **St = (r/dr) P(t) = 2k P(t)** (tensile hoop stress expression for an internally pressurized "thin walled cylinder," as found in most handbooks), and

***Axial*** (z-direction) Stress **Sz** = (Axial Force) / [(Inside Circumference) (Ring Thickness)]

$$Sz = [P(t)\ \pi r^2]\ /\ [(2\ \pi r)\ (dr)]$$

**Sz = (r/2dr) P(t)**

**Sz = St / 2** (handbook axial stress for closed-end, thin-walled cylindrical pressure vessels), and

**Sz = k P(t)**.

In terms of these three principal stresses, the effective von Mises tensile stress equivalent to the combined effect of our tri-axial stresses is:

$$Seff = \{[(Sr - St)^2 + (St - Sz)^2 + (Sz - Sr)^2]/2\}^{1/2}$$

A simple (uni-axial) ***tensile stress*** in the amount of **Seff** would produce the same amount of ***tensile strain distortion energy*** within a small element of a ductile material as do the ***three principal stresses*** acting together upon that same element. For any ductile metal such as our brass case neck material, we assume that ***no plastic strain*** can be caused by any reasonable amount or type of purely ***hydrostatic stress*** acting on a solid element of the metal. The concept of this *equivalent tensile stress* was developed simply to indicate how nearly the combined tri-axial stresses approach the point of *elastic failure* in materials such as steel. We can legitimately extend it here for use with "purely strain-hardening" materials like our example cartridge brass. We are making use of "von Mises equivalent tensile stress" calculations in the "elasto-plastic" stress region *above*, as well as in the purely elastic region *below*, the very low "yield point" of the annealed or partially hardened brass material of our cartridge case. The tensile stress required to produce a given amount of tensile strain always increases *monotonically* for these "well behaved" strain-hardening materials, both for *purely elastic* strains and for *combination*

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*elasto-plastic* strains. With a simple change in the constant value, we could instead work in term of *shear* stresses, strains, and distortion energies, if we so desired.

Substituting our symbolic solutions for these three *principal stresses* into this *von Mises equivalent tensile stress* relationship, and simplifying, we have:

$$\mathbf{Seff = P(t)\ [1 + 3k + 3k^2]^{1/2}}$$

And, for the particular neck tension stress of **19,146 psi** in our example, the chamber pressure at bullet release (**Pr**) for the brass of this ring-element at the rear of the neck can be found by setting the effective von Mises tensile stress **Seff** equal to our chosen example stress level for the annealed cartridge brass of the case neck and solving for the corresponding "release" chamber pressure **Pr** that will produce this amount of equivalent tensile stress:

**Seff = 19,146 psi**, and

$$\mathbf{Pr = Seff\ /\ [1 + 3k + 3k^2]^{1/2}}$$

**Pr = 19,146 psi / 10.40 = 1841 psi.**

This pressure level of **1841 psi** first occurs at about time **t = 75 µsec** (microseconds) into the pressure-versus-time curve for our example firing shown in Figure 2. Substituting this "release pressure" **Pr** back into the expressions for each of the three *principal stresses* at the instant of bullet release, we have:

**Sr = -Pr = -1841 psi**,

**St = 2k Pr = 20,251 psi**, and

**Sz = k Pr = 10,125 psi**.

The ***total*** normal force **Fn** gripping the straight walls of the bullet along up to **0.310-inches** of its body length would be:

**Fn = π (0.308 in.)(0.310 in.)(-Sr) = (0.300 sq. in.)(1841 psi) = 552 pounds**.

Assuming no "cold welding," and estimating a *coefficient of static friction* of not less than **0.4** between the bullet body and the case neck walls, the "bullet-pull" force would be at least **221 pounds**. At this chamber pressure of **1841 psi** the forward force **Fbb** on the base of the .308-caliber bullet at the instant of bullet release is:

**Fbb = (Bullet Base Area)(Pr) = (0.0745 sq. in.)(1841 psi) = 137 pounds**.

The estimated minimum **221-pound** "bullet-pull" at this neck tension would occur at a chamber pressure of **2966 psi**, or at about **100 µsec** in our example case. So we can see that our example bullet, seated with this **19 ksi** neck tension, could be ***released*** by the yielding of the case neck material at some time perhaps as early as **75 µsec**, but not much

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later than **100 µsec** when the bullet would be ***ejected*** anyway by the build-up of chamber pressure acting on its base**.**

I have measured between **500** and **1000 pounds** of axial force required under various conditions to engrave the rifling into the body of a match bullet with a soft lead core. After the release of the bullet, **Fbb = 500 pounds** when **P(t) = 6700 psi**, which occurs at **t = 160 µsec**, and **Fbb = 1000 pounds** at **P(t) = 13,400 psi**, which occurs at **t = 235 µsec** in our example pressure-versus-time curve. If not already in contact with the rifling, our **168-grain** bullet could freely travel **18.5 mils** forward during the **85 µsec** time interval from release to a force build-up to **500 pounds** of minimum possible engraving force. Or, it could jump **0.117-inch** during the **160 µsec** interval that must elapse before the full **1000 pounds** of maximal engraving force would be reached.

For a constant (or a time-averaged) acceleration **a** of the bullet, as determined from the average forward-acting force on the released bullet divided by its mass, the maximum free motion of the bullet is:

$$\text{Bullet Jump Distance} = \mathbf{s} = \mathbf{(a/2)\, t^2}$$

where **t** is used here as the *time interval*, in seconds, during which the bullet is free to move. I also used **7000 grains per pound** and **385.92 inches per second per second** as the acceleration of gravity in these calculations. The velocities (**a t**) with which the bullet impacts the chamber throat in these two extreme cases are also of interest: **36 feet per second** after jumping **18.5 mils**, and **122 feet per second** after jumping almost **1/8 inch**.

Thus, the released bullet will usually have enough time to ***stop momentarily in the chamber throat*** before pressure builds enough to engrave the rifling into its jacket material. This type of two-stage bullet motion cannot be conducive of top accuracy in a precision rifle which requires as nearly perfect alignment of the bullet axis with the bore as possible while the bullet is being engraved. Seating the bullet so that it contacts the rifling on loading is far the best way to prevent this “haphazard jamming” problem. Refer to my earlier article, “The Well Guided Bullet,” ***Precision Shooting***, September and October issues of 2006, for a full discussion of bullet alignment techniques.

## 3.0 Case Wall Expansion into Contact

Consider the case walls of our example ***precision reloaded, fire-formed cartridge*** to be comprised of a large number of ring-shaped elements of thin brass. These ***drawn brass*** case walls taper uniformly in thickness from over **45 mils** near the junction with the web of the case head down to a thickness of about **15 mils** just behind the shoulders of the case. The walls of our ***fire-formed case*** fit the chamber with only a *few mils of radial clearance*. The case walls have already expanded in previous firings and *need not plastically expand further here*. During ***precision reloading*** they were either:

1) Neck sized only,

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2) "Shoulder bumped" and neck sized, or
3) Minimally "Full Length" resized with a die incorporating a neck sizing bushing.

In any event, the headspace of the reloaded round should never exceed **1.0 mil**. This *minimum case wall clearance* caveat is a *major simplification* in the analyses and computations that follow, but it is justified here because it conforms to current "best practice" precision reloading procedures.

The hardness of the cartridge brass of the case walls also varies from approximately *quarter-hard* behind the shoulders to about *extra-hard* at the case web. We will work out the yielding of the ring of brass located just behind the shoulders of the cartridge that *first yields* into contact with the chamber walls as pressure begins to rise. Then we will examine a ring-shaped element back at the "**0.280-inch** point" where the *last yielding* of the case walls into contact with the inside walls of the chamber takes place.

### 3.1 At the First Point of Interest

At the location just behind the shoulders of the case where the brass case walls *first yield* into contact with the chamber walls:

| | | |
|---|---|---|
| ID of match chamber at shoulder | | = **0.455 inch** |
| Case wall thickness | = **dr** | = **0.015 inch** |
| Case (ring) ID at contact = **2r** | | = **0.425 inch** (for fire-formed cases) |
| Let the ratio | **k = r/2dr** | = **7.0850** (at this location). |

And once again, the effective von Mises tensile stress **Seff** can be expressed in terms of the usual three principal stresses at this location as:

$$\mathbf{Seff = \{[(Sr - St)^2 + (St - Sz)^2 + (Sz - Sr)^2]/2\}^{1/2}}$$

But the principal stresses in this brass ring-element are not necessarily the same functions of chamber pressure **P(t)** as they were previously when we were looking at a ring-element of the case neck. The *radial* and *tangential* stresses can be parameterized just as before:

**Sr = -P(t),** and

**St = 2k P(t).**

However, the *axial* (z-direction) stress **Sz** in the case walls must be *reduced* because of the presence of the primer cup acting as a smaller, secondary piston within the case head and acting separately against the bolt face. This primer force itself *does not* contribute to the stretching of the case walls back toward the bolt face, but the force of friction between the primer cup and the walls of its pocket ***resists any relative motion***, including the "pushing out" of the primer cup against the bolt face while the case head is being

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"held back" by the larger "wall stretching" forces. This primer friction force is *dragging forward on the case head*, and so it *does* contribute additively to case wall stretching.

Looking only at the *peak forces* occurring at the maximum chamber pressure **Pm = 57,400 psi**, let us define the *fraction* of the total peak force on the case head **Ft(Pm)** that is available to *stretch the side walls of the case* to be another ratio **b**, such that:

$$\mathbf{b = [Ft(Pm) - Fppr(Pm) + Ffpr(Pm)] / Ft(Pm) = \underline{0.8010}}$$

where **Ft(Pm)** = Total peak rearward force on case head assembly

$$\mathbf{= Pm\ (\pi/4)\ D^2 = 6282\ pounds}$$

**Fppr(Pm)** = Peak potential primer force on bolt face

$$\mathbf{= Pm\ (\pi/4)\ d^2 = 1988\ pounds}, \text{ and}$$

**Ffpr(Pm)** = Peak primer sidewall friction force in the primer pocket

$$\mathbf{= (Cf)\ (Pm)\ (Apw) = 738\ pounds.}$$

where **d = 0.210 inch**, the outside diameter of a "large rifle" primer cup, and
**Cf = 0.30 =** Coefficient of friction used in the FEA for the primer
cup in the primer pocket of the case head**.**

Here, we are using the *adjusted value*, **D = 0.3733 inch**, for the ***effective piston diameter*** inside the case head of our example .308 Winchester cartridge. This value was derived from the proportional relationship between the two effective piston areas involved (and thence the squares of their diameters) and their peak bolt thrust forces at peak chamber pressure:

$$\mathbf{D = d\ [Ft / Fppr]^{1/2} = d\ [(Fb + Fz) / (Fpr + Ffpr)]^{1/2} \approx d\ [Fb / Fpr]^{1/2}.}$$

So, $\mathbf{D = (0.210\ inch)\ [3950\ pounds / 1250\ pounds]^{1/2} = \underline{0.3733\ inch}}$

where **Fb** and **Fpr** are ***peak total and primer bolt thrust values, respectively,***
***scaled*** ***from FEA output plots***, and

**Fz** is the ***calculated*** peak axial "wall stretching" force of **2332 pounds**.

By setting both: **Fb** **= q Ft** **= q [Fb + Fz]** and

**Fpr** **= q Fppr** **= q [Fpr + Ffpr]** simultaneously,

In effect we are simultaneously setting both:

$$\mathbf{Fz = (1 - q)\ Fb = (1 - q)\ (3950\ pounds) = \underline{2332\ pounds}}, \text{ and}$$

$$\mathbf{Ffpr = (1 - q)\ Fpr = (1 - q)\ (1250\ pounds) = \underline{738\ pounds}}.$$

Only with **q = 0.4096**, can both **Fz** and **Ffpr** assume these *reasonably explainable* peak values *simultaneously*. [The non-zero constant **q** divides out of the above expression for **D** and is not used again hereafter.]

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We are also *adjusting* the inside depth of the primer cup to be **<u>0.0758-inch</u>** so that:

**Ffpr = (Cf) (Pm) (Apw) = Fppr – Fpr = 1988 pounds – 1250 pounds = 738 pounds**

with **Apw = π (0.180 inch) (0.0758 inch) = 0.042864 square inches**.

[I measured several fired "large rifle" primer cups of Federal and other unknown makes and found them to vary widely in inside depth from as shallow as about **0.075 inches** to almost **0.090 inches** in interior depth. We install the primers so that their protruding anvils "bottom out" (with a uniform *additional* seating pressure) in primer pockets that have been uniformed to some chosen depth between **0.128-inch** and **0.132-inch**. The heads of the seated primers usually run about **6** to **8 mils** below flush with the case heads. The heads of the primer cups measure about **0.030-inch** in thickness. The fired primer cups seem consistently to measure right at **0.180-inch** in inside diameter, and wall thickness is consistently **15 mils**.]

So, getting back to our principal stresses, and asserting that the fraction **b** is likely to keep its same value away from the pressure peak; our *adjusted* axial tensile stress is now given by:

**Sz = bk P(t).**

Then, after a little algebra, we find that *anywhere within the case walls*, starting just behind the shoulders and extending aft to the web of the case head, the effective von Mises tensile stress is:

$$\mathbf{Seff = P(t)\ [1 + (2 + b)k + (4 - 2b + b^2)k^2]^{1/2}}$$

$$\mathbf{Seff = P(t)\ [1 + (2.8010)k + (3.0396)k^2]^{1/2}}$$ (This should hold for any cartridge using "large rifle" primers in a ".308-size" case head.)

Recalling that this portion of the fire-formed case was plastically expanded with the chamber in firing, and then "sprang back" elastically to its current shape, we can set **Seff = 36,400 psi**, the estimated uni-axial "***yield point"*** tensile stress for *quarter-hard cartridge brass*, and then calculate the particular contact pressure (**Pc**) required to cause the yielding of this ring into *first contact* with the inside chamber walls:

$$\mathbf{Pc = Seff / [1 + (2.8010)k + (3.0396)k^2]^{1/2} = 36{,}400 / 13.169 = 2764\ psi.}$$

This rather low chamber pressure first occurs at $\mathbf{t_1 = 90\ \mu sec}$ into our example pressure-versus-time curve.

Now, we can substitute this *contact pressure* (**Pc**) back into the expressions for the principal stresses at this location in the brass ring-element of the case walls just behind the shoulders at the instant of yielding into contact:

**Sr = -Pc = -2764 psi**

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**St = 2k Pc = 39,166 psi**

**Sz = bk Pc = 15,686 psi.**

And now we can calculate the (maximum) amount of ***purely elastic*** tangential (hoop) strain **Th** that can be stored in this *fire-formed* ring-shaped element of "quarter-hard" brass by dividing the "yield point" tangential stress **St(Pc)** by Young's Modulus of Elasticity **E**:

**Th = (39,166 psi) / (16 msi) = 0.245 percent.**

Interestingly, the peak radial-direction expansion (**1.1 mils** in diameter) shown in the fully dynamic FEA study for the ***barrel steel*** immediately surrounding this portion of the chamber (**ID = 0.455 inch**) divides out to an elastic strain rate of **0.242 percent**. [We had previously calculated (by applying Lamé's Equations to the bare chamber walls) that a hydrostatic internal pressure equal to our example peak chamber pressure here would result in an increase of **1.5 mils** in the inside diameter at this location in the steel immediately surrounding the shoulder of the chamber *with no brass cartridge case inside*.] Therefore, both this ring of brass material and the steel surrounding (and in contact with) this brass will "spring back" at ***closely matching rates*** when the chamber pressure drops later, just so long as this brass material is *at least "quarter-hard."* If this brass case wall material immediately behind the case shoulders were accidentally to become *annealed*, the resulting **75-percent smaller** maximum purely elastic strain rate could cause the case shoulders to "stick" as the steel chamber walls shrink radially with falling chamber pressure. [This likely explains the "shoulder sticking problem" many of us have experienced with wildcat designs based on the WSSM cartridge brass. These short, fat cases behave as if their walls might be ***too soft***, as well as being ***too thick*** at **25 mils**, in the region just behind the shoulders.]

### 3.2 At the Second Point of Interest

Now, let us make the same calculations for the *last-yielding* ring of thicker, presumably *extra-hard* brass contacting the chamber walls at the point **0.280 inches** (plus or minus about **0.010-inch**) from the case head. Examination of many once-fired cases from many different manufacturers, as well as many multiply-reloaded cases, all with ".308-size" case heads, confirms that the thicker (more than **0.045 inches** thick), stronger (probably extra-hard) brass of the case walls, head and web behind this "**0.280-inch** point" ***never*** expands into contact with the chamber walls during normal firing.

| | | |
|---|---|---|
| Match chamber ID at "**0.280-inch**" point | | **= 0.470-inch** |
| Brass case wall thickness | = **dr** | **= 0.045 inch** |
| Case ID at contact | = **2r** | **= 0.380 inch** |
| So, here the ratio | **k = 2r / 4dr** | **= 2.111** |

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[Any **k-value** of less than **2.5** indicates that we are extrapolating the implicit "thin walled cylinder" formula into the region in which, by rule of thumb, the full version of Lamé's equations should be used instead to evaluate stress as a function of radial position within the "thick walled cylinder." We continue to use the "thin wall" relationship here for algebraic tractability.]

If we now set the effective von Mises stress equal to **59,200 psi**, the estimated yield point tensile stress for *extra-hard brass,* we can again solve for the particular chamber pressure **Pc** necessary to yield this last part of the brass case walls into full contact with the inside walls of the chamber:

$$\mathbf{Pc = Seff / [1 + (2.8010)k + (3.0396)k^2]^{1/2}}$$

$$\mathbf{Pc = 59{,}200 / 4.523}$$

$$\mathbf{Pc = 13{,}088\ psi.}$$

This still-relatively-low chamber pressure occurs at time $\mathbf{t_2 = 230\ \mu sec.}$ into our example pressure curve.

So, the three principal stress values within the brass ring-shaped element of the case walls at this "**0.280-inch** point" at the instant of yielding into full radial contact with the inside of the chamber walls, are:

$$\mathbf{Sr = -Pc = -13{,}088\ psi}$$

$$\mathbf{St = 2k\ Pc = 55{,}260\ psi}$$

$$\mathbf{Sz = bk\ Pc = 22{,}132\ psi.}$$

Keep in mind that the example case has already been plastically shaped by *fire-forming* to match the inside of the chamber and has only been *precision reloaded* thereafter.

## 4.0 Elastic Stretching of the Brass Case Walls

Now, *assuming sufficient headspace clearance* **H**, we can calculate the total amount of ***elastic elongation*** (or linear "stretch") that would be "locked into" portions of the case walls as they yield into contact with the steel chamber walls at these relatively low pressures. The initial headspace **H** is **2.1 mils** in our FEA study. ***The axial tensile stress acting across each ring-shaped element of the brass case walls, as it is being "laid down" freely into contact with the chamber walls, is attributable only to the chamber pressure that caused the yielding of that element into that contact***.

At our *first point of interest* just behind the case shoulders, at $\mathbf{z_1 = 1.554}$ **inches** from the case head, the ***elastic strain*** $\mathbf{Tz_1}$ in the ***axial*** (**z**) ***direction*** across the ring element at this location and that will be "locked into" the ring of brass shortly after *first contact* can be found via Hooke's Law from the ***axial stress*** at this location $\mathbf{Sz(z_1)}$ due to the rather surprisingly low chamber pressure of first yielding into contact of only **2764 psi**:

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$$Tz_1 = Sz(z_1) / E$$

$$Tz_1 = (15{,}686 \text{ psi}) / (16{,}000{,}000 \text{ psi})$$

$$Tz_1 = 0.0980 \text{ percent}$$ (a *purely elastic* axial strain),

where Young's Modulus of Elasticity (**E**) for cartridge brass is **16,000,000 psi**. This elastic strain rate is itself only **43 percent** of the maximum purely elastic *uni-axial* strain of **0.228 percent** sustainable by this "quarter-hard" cartridge brass material.

And, similarly, at $z_2 = 0.280$ **inches** from the case head, the ***elastic axial strain*** $Tz_2$ due to the still low chamber pressure of **13,088 psi**, and that would naturally be "locked into" the last portion of the case to yield fully into contact with the chamber walls, can be found from the axial stress at our *second point of interest* $Sz(z_2)$ to be:

$$Tz_2 = (22{,}132 \text{ psi}) / (16{,}000{,}000 \text{ psi})$$

$$Tz_2 = 0.1383 \text{ percent}$$ (also an elastic strain).

This axial strain is about **37 percent** of the maximum purely elastic strain rate of **0.370 percent** for the presumed extra-hard brass material involved.

As a first approximation we can assume that the "locked in" strain rates *vary linearly with* **z** over the varying hardness and thickness of the brass walls between these points at the two ends of the brass case-wall contact region, so that the average strain rate would be **0.1182 percent** over this range in **z**. And the change in headspace (**dH**) that would be ***elastically absorbed*** by and ***stored*** in these case walls expanding *freely* into contact with the chamber at these low pressures would be found to be:

$$dH = (z_1 - z_2)\,(0.000980 + 0.001383) / 2$$

$$dH = (1.274 \text{ inches})\,(0.001182)$$

$$dH = \underline{1.506 \text{ mils}}.$$

Whatever is the exact value of **dH** for this example situation, an effective initial headspace **H** of less than that amount would merely result in *earlier case head contact* during the sequential process of the yielding of the case walls and in *less cumulative axial strain* being "locked into" the brass case walls as they are "laid down" into contact with the chamber walls, without slipping, over this relatively low pressure range of **2.76 ksi** to **13 ksi**. With our typical example pressure-versus-time curve, the time interval that elapses between these first and last wall contacts is only $t_2 - t_1 = 120\ \mu sec$, out of a total simulated firing-event duration of **1025 µsec**.

Whenever there is some non-zero initial headspace **H**, but *less* than **dH**, the *same pressure-dependent elastic strain rates* will always be "locked into" the brass during the *initial interval* starting at the time $t_1$ of first wall contact, and continuing until *case head contact* at time $t_c$. During the entire remainder of the chamber pressure excursion after $t_c$,

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the axial strain rates in the brass that has not yet contacted the case walls will depend primarily upon the "stiffness modulus" for the locked bolt of the rifle's action and secondarily upon the friction characteristics of the chamber walls. For our example blueprinted Remington 700 action, its "effective dynamic stiffness modulus" is **4300 pounds** of peak bolt thrust **per mil** of maximum bolt-face setback in this fully dynamic FEA study, or **4.3 million pounds per inch** of setback. [In an earlier ***static*** FEA study, the stiffness modulus for this same bolt action had been found to be **2.9 million pounds per inch** for hydrostatic chamber pressures. Each value is correct when used appropriately.]

## 4.1 Elastic Stretching of the Case Head

After all portions of the brass walls that will ever yield *have yielded* into chamber contact, the steel walls of the chamber support the brass case walls very well. The stress situation within the case walls in this "excess headspace" situation of our example here then simplifies from ***three principal stresses*** being involved down to just a ***two-axis stress*** internally pushing the case head back toward contact with the bolt face:

$$\mathbf{Sr = -P(t),}$$

$$\mathbf{St = 0}$$

$$\mathbf{Sz = bk\ P(t).}$$

So now the effective von Mises tensile stress here in the contacting brass case walls at the "**0.280-inch** point" becomes:

$$\mathbf{Seff = P(t)\ [1 + bk + b^2k^2]^{1/2} = P(t)\ [2.3560]}$$

Once again setting **Seff =59,200 psi**, the estimated yield point where production of plastic strain can begin for the extra-hard brass of the case wall region around the "**0.280-inch** point," and solving for this particular "secondary" axial yield pressure **Pa**:

$$\mathbf{Pa = (59{,}200\ psi) / (2.3560) = 25{,}128\ psi \approx 25\ ksi}.$$

So, now we can begin to see how P. O. Ackley was able to get away with firing full-load .30-30 Winchester cartridges in an old Model 94 lever-action with its locking bolt removed. In our example FEA study, the case head *first contacts the bolt face* at a chamber pressure of about **25 ksi** at **280 μsec**, which (perhaps not coincidentally) happens also to be the chamber pressure at which the case walls will finally *yield axially* even with good radial support from the chamber walls.

Our critical z-axis stress **Sz** at **Pa = 25 ksi** for beginning the ***plastic stretching*** of the case lengthwise, mostly within a narrow region centered on the "**0.280-inch** point," is:

$$\mathbf{Sz(25\ ksi) = bk\ Pa = (0.8010)\ (2.111)\ (25{,}128\ psi) = 42{,}491\ psi}$$

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Even after the case head contacts the bolt face, they ***continue moving rearward together*** toward the maximum bolt face setback at peak chamber pressure **Pm = 57,400 psi**.

Starting at the secondary "yield pressure" **Pa** of **25 ksi**, this axial stress **Sz** can increase by about **10 percent** during the first **0.5 percent** of ***plastic strain*** occurring at higher subsequent chamber pressures. The axial stretching force **Fz(25 ksi)** corresponding to this axial stress **Sz(25 ksi)** can be estimated by multiplying this axial stress **Sz** value by the cross-sectional *area* of the "back edge" of this ring-shaped brass element at the "**0.280-inch** point:"

**Fz(25 ksi) = [Sz(25 ksi)] [Area of "back edge" of last ring-element to yield]**

**Fz(25 ksi) = (42,491 psi) [π (0.380 inch) (0.045 inch)]**

**Fz(25 ksi) = 2283 pounds**.

[Here, I chose to use the inside diameter of the case walls (**0.380 inch**) instead of the perhaps more appropriate "mid-line" diameter (**0.425 inch**) simply to avoid calculating the "too-large" value of **2553 pounds** for **Fz(25 ksi)**. This inconvenience is probably another ramification of the different results obtained from *fully dynamic* versus *quasi-static* analyses.]

At the subsequent peak chamber pressure **Pm** of **57.4 ksi**, occurring at **475 μsec** in this example, the total potential force **Ft(Pm)**, acting on the entire case head assembly, was previously calculated to be:

$$\mathbf{Ft(Pm) = (57.4\ ksi)\ [(\pi/4)\ (0.3733\ inch)^2] = 6282\ pounds}$$

But, this calculated total potential force **Ft(Pm)** should equal the sum of the actual (from the FEA) **3950-pound** bolt thrust at peak chamber pressure **Fb(Pm)** plus the calculated case wall stretching force **Fz(25 ksi)**, increased by a *calculated adjustment* of **2.16 percent** to correspond to this peak pressure level, so that the value of **Fz** at peak chamber pressure is:

**Fz(57.4 ksi) = 6282 pounds – 3950 pounds = 2332 pounds**, or

**Fz(57.4 ksi) ≈ (1.0216) Fz(25 ksi)**.

## 5.0 Explanation of the FEA Outputs of Peak Bolt Thrust Values

As plotted in Figure 4, below, the peak bolt thrust **Fb(Pm)** of **3950 pounds** is itself the sum of two component peak forces: **1250 pounds** of peak primer contact force **Fpr(Pm)** and **2700 pounds** of peak case-head contact force **Fch(Pm)** (as each force is *summed over all contacting surface elements* and plotted by LS-DYNA).

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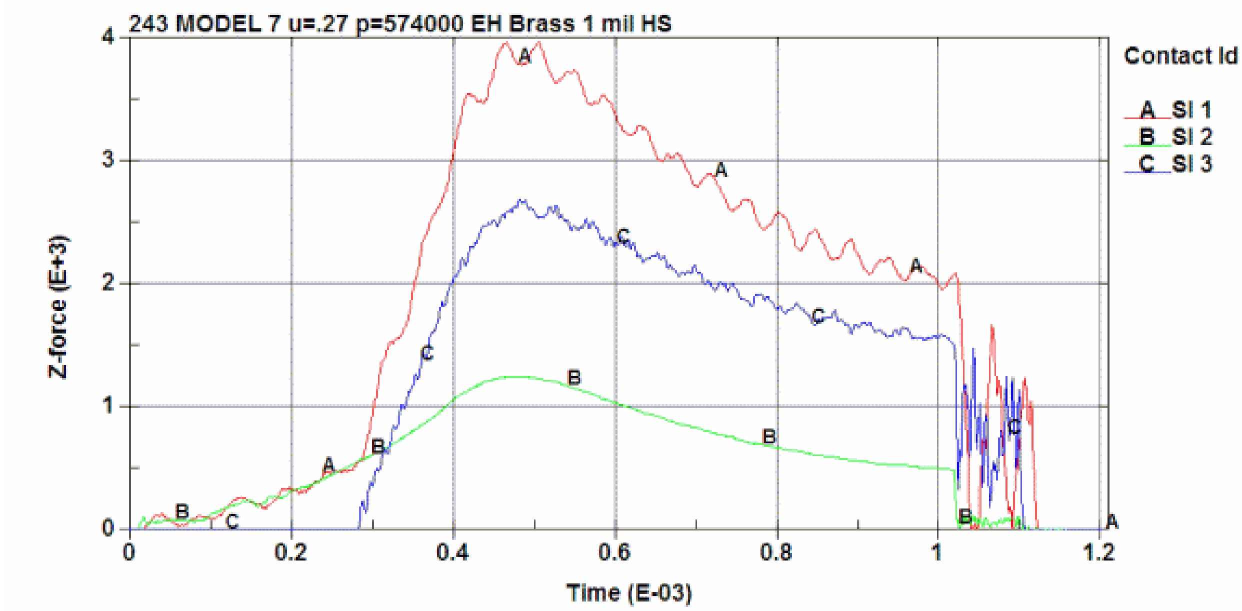


KEY: Curve **A** (Red) is the **total bolt thrust Fb(t)** in pounds.

Curve **B** (Green) is the **primer cup contact force Fpr(t)** in pounds.

Curve **C** (Blue) is the **case head contact force Fch(t)** in pounds.

And **Fb(t) = Fch(t) + Fpr(t)**.

**Figure 4. Summed Contact Forces at Bolt Face vs. Time as Plotted by LS-DYNA**

We can explain the **1250-pound** primer contact force **Fpr(Pm)** pretty well as the geometric potential primer force at this pressure **Fppr(Pm)** ***minus*** the force of friction **Ffpr(Pm)** between the primer cup and the inside walls of its primer pocket:

**Fppr(Pm) = (57.4 ksi) [(π/4) (0.210 inch)$^2$] = 1988 pounds**

**<u>-Ffpr(Pm) = -(0.30) (57.4 ksi) [π (0.180 inch) (0.0758 inch)] = -738 pounds</u>**

**Fpr(Pm) = Fppr(Pm) – Ffpr(Pm) = = 1250 pounds.**

As mentioned earlier, the "effective depth" for the primer cup used here, **0.0758-inches**, is a reasonable value, *adjusted* to produce exactly the required friction force at this peak pressure. While the primer pocket is *contained within* the case head, and even though both parts are assumed always to experience the *same internal pressure levels (as they do in the FEA model)*, the correction for friction is *subtractive* to the primer contact force here because *only the case head itself is being held back* by the axial case-wall stretching force **Fz**.

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And the **2700-pound** peak force of case-head contact **Fch(Pm)** can be explained as the algebraic sum of the peak potential case-head force **Fpch(Pm)** (from geometry), ***minus*** the peak case head stretching force **Fz(57.4 ksi)**, ***plus*** the peak force of primer friction **Ffpr(Pm)** as estimated above:

| | | |
|---|---|---|
| **Fpch(Pm)** | **= (57.4 ksi) [(Area of case head) - (Area of primer pocket)]** | |
| | **= 6282 pounds – 1988 pounds** | **= 4294 pounds** |
| **-Fz(57.4 ksi)** | | **= -2332 pounds** |
| **+Ffpr(Pm)** | | **= + 738 pounds** |
| **Fch(Pm)** | | **= 2700 pounds,** |

We can write several different symbolic expressions for the total peak force **Fb(Pm)** exerted on the bolt face (that is, the "bolt thrust") at peak chamber pressure (**Pm = 57,400 psi**):

**(1) Fb = Fpr + Fch**

**= 1250 pounds + 2700 pounds**

**= 3950 pounds** (all FEA plotted results).

**(2) Fb = [Fppr – Ffpr] + [(Fpch) – Fz + Ffpr)]**

**= [1988 – 738 pounds] + [(6282 – 1988 pounds) - 2332 + 738 pounds]**

**= [1250 pounds] + [2700 pounds]**

**= 3950 pounds** (compared to above), and

**(3) Fb = Fppr + Fpch – Fz**

**= 1988 pounds+4294 pounds– 2332 pounds**

**= 3950 pounds** , and finally

**(4) Fb = Ft – Fz**

**= 6282 pounds – 2332 pounds**

**= 3950 pounds**.

Keep in mind that these "peak pressure" relationships, while important because of the larger forces involved, are actually occurring only *very briefly*, and are significantly influenced by ***transient inertial factors***. In other words, these relationships *might not necessarily translate directly* into other, less dynamic, lower-pressure regimes.

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## 5.1 Analysis of the Bolt Thrust Forces

We could attempt to define a computational procedure that would generate the curve **Fb(t)** based on the chamber pressure curve **P(t)** and on the analytical expressions developed above. There would be myriad special cases:

1) Pressure (or time) breakpoints at **0.0**, **2.76 ksi**, **13 ksi**, **25 ksi** and **57.4 ksi** on just the *rising arm* of the **P(t)** curve,
2) Separately treated *initial headspace* cases of **zero**, less than **1.5 mils**, between **1.5 mils** and about **6.0 mils**, and greater than **6.0 mils**,
3) Weak versus strong actions, trued versus un-trued actions, etc.

[We have just outlined a total of 64 pressure intervals and special cases, each requiring a separate analysis.]

But, more basically, in any situation where the *case head has not yet contacted the bolt face*, the bolt thrust **Fb(P)** is simply the ***primer thrust minus the force of friction in the primer pocket***. Each of these forces has been formulated analytically as *linear functions of the chamber pressure* **P(t)**:

**Fb(P) = Fppr(P) – Ffpr(P)**

**Fb(P) = P(t) (π/4) $(0.210\text{inch})^2$ – (0.30) P(t) [π (0.180 inch) (0.0758 inch)]**

**Fb(P) = (0.021777 square inches) P(t).**

Due to the low mass of the primer, this "peak pressure" relationship should also hold fairly well at lower pressures.

We can also calculate a partial amount of the bolt face setback **SBp(P)** corresponding to any hydrostatic chamber pressure **P(t)** to add into the original headspace **H** to form what we could term the "effective headspace" value, as long as the case head has not yet contacted the bolt face:

**SBp(P) = (**Hydraulic primer force minus primer friction force) / (Bolt static stiffness modulus),

**SBp(P) = Fb(P) / Mbs**

**SBp(P) = [(0.021777 square inches) P(t)] / (2.9 x $10^6$ pounds per inch)**

**SBp(P) = (7.5093 x $10^{-9}$ inch) P(t)**.

In this example, this partial bolt face setback *due only to the net primer thrust* could never exceed **0.291 mils**, that would corresponding to the maximum possible **1250 pounds** of net *dynamic* bolt thrust from contact with the primer alone at peak chamber pressure **Pm**:

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**SBp(Pm) = (1250 pounds) / (4.3 million lbs/in) = 0.291 mils.**

Then, *after contact of the case head with the bolt face*, we can forget about the separate primer thrust and formulate the bolt thrust as the total potential case head force **Ft(P)**, ***minus*** the stretching force **Fz(P)** acting on the case walls.

**Fb(P) = Ft(P) – Fz(P)**, where

**Ft(P) = P(t) [(π/4) (0.3733 inch)$^2$]**, or

**Ft(P) = (0.1094475 square inches) P(t)**, and

And, the highly *non-linear function* **Fz(P)** must pass through the points:

**Fz(P<2.76 ksi) = 0.0,**

**Fz(13 ksi) = 400 pounds,**

**Fz(25 ksi) = 2283 pounds**, and

**Fz(57.4 ksi) = 2332 pounds**.

Above **25 ksi**, the stretching force apparently becomes *nearly constant*, slowly rising from **2283 pounds** to **2332 pounds** at **P(t) = 57.4 ksi**.

The peak dynamic setback of the bolt-face **SB(Pm)** in the LS-DYNA FEA study is **0.92 mils**. The FEA study also shows that at the moment of case head contact, the total bolt thrust **Fb(25 ksi)** is about **600 pounds**. Then the ***bolt-face setback*** **SB(25 ksi)** should be found from:

**SB(25 ksi) = (600 pounds) / (2900 lbs/mil) = 0.21 mils**.

Therefore, the remaining portion of the bolt-face setback **SB** above **25 ksi** must be:

**SB(25 ksi<P<Pm) = SB(Pm) – SB(25 ksi)**

**SB(25 ksi<P<Pm) = 0.92 mils – 0.21 mils = 0.71 mils.**

The bolt setback that occurs during the chamber pressures rise from **25 ksi** to **57.4 ksi** is *significant* because, since it only happens *after* the case walls have "gone plastic" at the **0.280-inch** point, this entire amount (**0.71 mils)** turns into ***plastic axial elongation*** within a narrow ring at this location in the case walls.

## 6.0 Case Wall Normal Contact Forces

When the effective headspace **H** of a cartridge fired in our example rifle exceeds **dH = 1.5 mils**, as it does in our example here, the ring-elements of the case walls keep successively "laying down" into contact with the chamber walls until the last yielding of the case walls takes place (at the **0.280-inch** point) as chamber pressure reaches **13 ksi**. At this instant, the *unevenly distributed* contact forces between the brass case walls and the steel walls of the chamber can be crudely modeled as varying *linearly* with the

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longitudinal **z-**dimension from a minimum of **zero** back at the recently contacting "**0.280-inch**" end of this ***tapered brass cylinder*** to a maximum force of *double the average force per unit surface area* up at the shoulder end. And the *average* distributed normal-direction force at this pressure **Fn(13 ksi)** is approximately:

$$\mathbf{Fn(13\ ksi) = (1/2)\ (13\ ksi - 2.76\ ksi)\ Afc = 9{,}560\ pounds},$$

with **Afc** = **Area of the Frustum of the Cone**

$$= (\pi/2)\ (0.470\ \text{inch} + 0.4551\ \text{inch})\ [(1.2743\ \text{in.})^2 + (0.25)\ (0.470 - 0.4551)^2]^{1/2}$$

$$\approx (\pi/2)\ (0.470\ \text{inch} + 0.4551\ \text{inch})\ (1.2743\ \text{inch})$$

$$= 1.8517\ \text{square inches.}$$

For a clean, dry case in our "crocus cloth" polished chamber, the ***coefficient of sliding friction* Cf** between the case and chamber walls is only **0.27**, as measured by Dick Hatfield. Even so, the "z-averaged" ***force of friction*** "locking" the case and chamber walls together is

$$\mathbf{(Cf)\ (Fn(13\ ksi)) = 2581\ pounds}$$

in this *polished chamber* at a chamber pressure of **13 ksi**.

Also at this pressure, the total axial force **Ft(13 ksi)** of the case head pulling rearward on these case walls is:

$$\mathbf{Ft(13\ ksi) = (13\ ksi)\ (\pi/4)\ (0.3733\ inch)^2 = 1432\ pounds}.$$

Then, while the chamber pressure is rising from **13 ksi** to the "secondary axial yield pressure" **Pa** of **25 ksi** for the case walls in contact with the chamber walls forward of the "**0.280-inch** point," the case walls ***stretch elastically*** and ***slide rearward*** along the chamber walls in a complicated process that I *cannot analyze*. At the beginning of this pressure interval (at **P = 13 ksi**), the effect of the **1432-pound** axial force stretching the "thick end" of the brass cylinder is ***naturally spread across the entire* 1.274-inch-*long cylinder***. At the beginning of this interval, there is **zero** friction force to "lock" the case walls in place back at the (just now contacting) stronger and thicker "case head attachment" end of the cylinder, but the resisting friction force is ***concentrated up front*** at the much weaker "thin end" of the cylinder. ***This would seem to be exactly the situation in which the coefficient of friction* Cf *between the case and the chamber walls would be a more important chamber design variable than during any other interval in the expansion phase of the firing process.***

During the **12 ksi** pressure increase from **13 ksi** to **25 ksi** in this example, the normal contact force ***increases tremendously*** by an additional **dFn**:

$$\mathbf{dFn = (12\ ksi)\ (Afc) = 22{,}220\ pounds}.$$

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The "z-averaged" normal contact force increases to:

**Fn(25 ksi) = 31,780 pounds**.

And the total force of friction increases to:

**(Cf) (Fn(25 ksi)) = 8,581 pounds,**

while the stretching force only increases to:

**Ft(25 ksi) = 2736 pounds.**

***Two significant events occur before the chamber pressure reaches the secondary (axial) yield pressure of* 25 ksi:**

1) ***The sliding of the case walls is halted by the appearance of this huge, more evenly distributed normal force together with its resulting friction force, and***
2) ***The elastic stretching of the case walls becomes concentrated into the region immediately surrounding the*** "**0.280-inch**" ***point at the rear of this contacting brass cylinder.***

## 7.0 Plastic Strain in the Case Walls

As the "irresistible force" stretching the case-head rearward eventually rises to a peak of **6282 pounds** and meets the "immovable object" of up to **28,700 pounds** of static friction at peak chamber pressure **Pm**, is it any wonder that these case walls will suffer ***plastic stretching*** at the "**0.280-inch** point," after yielding axially in that area at a pressure of only **25 ksi**? ***All plastic strain*** occurs within a narrow region about **40** to **50 mils** in width centered upon the "**0.280-inch**" point. The amount of elongation occurring here is ***whatever headspace and bolt face setback remain*** after the chamber pressure reached **25 ksi**. The *specified maximum elongation* for "full-hard" cartridge brass is **8 percent**, and for "extra-hard" cartridge brass, it is only **5 percent**. **Eight percent** of **50 mils** is only **4 mils** and **5 percent** of **40 mils** is only **2 mils**, so ***both the initial headspace and the bolt-face setback "stiffness modulus" are critical in preserving the structural integrity of our cartridge cases.*** Cases which have been severely stretched within this region will often show a band of shiny brass around the outside of this part of the case. For the oblivious reloader, this band of shiny material around the "**0.280-inch**" point often constitutes the final warning of incipient catastrophic case head separation. [All of this is why I have been writing for many years that we must always maintain the ***total headspace*** of our precision reloads to be no larger than **one mil** to avoid unnecessarily damaging our brass cases.]

## 8.0 Summary

The annealed brass of the case neck yields and consequently releases the bullet at some chamber pressure between **800** and **2500 psi**, depending on the amount of case neck expansion that had occurred during bullet seating. If the bullet had not been seated out

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into contact with the throat of the rifling, it will stop there awaiting the build-up of enough chamber pressure to engrave the rifling into its body (between about **6.7** and **13.4 ksi**). Between the first yielding of the case walls at **2.8 ksi** and the last at **13 ksi**, the case walls will "lock in" and elastically store **1.5 mils** of axial elongation, provided the headspace allows this much movement without contacting the bolt face. The initial headspace was **2.1 mils** in this example. Between chamber pressures of **13 ksi** and **25 ksi**, where the case walls back at the "**0.280 inch"** point finally yield axially, a further **0.8 mils** (in this example) of elastic elongation is spread over and "locked into" these contacting case walls in a ***highly complex and highly variable process*** involving both ***elastic elongation*** and ***differential sliding*** of the case walls in contact with the chamber. This is the *only* portion of the cartridge expansion phase in which the ***coefficient of sliding friction*** between the case walls and the chamber walls is a significant variable. And this friction is easily modified by the presence of bore oil, case lube or bore cleaning solvent remaining in the chamber upon firing. After reaching **25 ksi**, any remaining headspace (none in our example here) and all remaining bolt-face setback (an additional **0.7 mils** here, at peak pressure) will begin to be converted into ***plastic elongation*** within a narrow region centered on the "**0.280 inch**" point. A total of **2.3 mils** of elastic elongation is stored in the entire contacting region of the case walls in this example by the time of peak chamber pressure, and **0.7 mils** of plastic stretching has occurred in one small area of these case walls, for a total of **3.0 mils** of case stretching during firing. In this example, all of the plastic stretching distance of **0.7 mils** originated as bolt-face setback occurring at chamber pressures above **25 ksi**. The total bolt-face setback in this example is **0.92 mils**. The ***post-firing headspace*** for our example .308 Winchester cartridge case fired in our example chamber will likely equal the original headspace of **2.1 mils** minus the **0.7 mils** of *permanent plastic elongation* of the case walls, for a net **1.4 mils** of post-firing headspace. We should next examine what happens in detail to our brass cartridge case as the chamber pressure drops after its peak and as the bolt-face returns forward elastically toward its original position.

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