Vibration Control in Bolt Action Rifles

Part I

By Jim Boatright

Introduction

The control of vibrations and recoil motions that arise during the firing of a rifle is of critical importance in delivering the bullet to the target accurately. This article addresses the mechanical motions of the rifle and its component parts from the moment the sear breaks due to trigger pressure until the bullet exits the crown of the barrel and clears the muzzle blast shock wave. The rifle is assumed to be in a fully supported, stationary, horizontal firing position as in benchrest competition. While some of the ideas discussed here may have application to semi-automatic rifles or to rimfire rifles, the precision centerfire bolt action rifle is the primary subject of these discussions. This article is the third in a series dealing with various aspects of designing and building "seriously accurate" bolt action rifles. The section here in Part I on the vibration effects for standard rifle barrels upon bullet trajectories contains a modicum of "new work." Others have separately treated the "launch angle error" effect of barrel vibrations or the "cross-track velocity kick" effect as if either were the sole cause of the variations in group sizes and locations observed during barrel tuning, but here we show how they are actually inseparable and always occur together. We cannot adequately deal with this topic without resorting to the use of some basic mathematics and physics. No animals, nor math-averse humans, were injured in the production of this article.

Even quite small lateral motions and variations in the pointing direction of the muzzle at the time the bullet exits the barrel will produce significant displacements of the bullet impacts on a distant target. While there are many types of vibrations occurring in a rifle being fired, we will limit our concern here to transverse standing wave vibrations in a vertical plane affecting the muzzle of the rifle barrel. We do this because these vibrations are the ones that seem most persistently to affect rifle accuracy. Other types of vibrations are not addressed because either: 1) they cannot mechanically affect rifle accuracy, 2) with good rifle building and shooting techniques, they just do not seem to cause accuracy problems, or 3) they may be corrected simultaneously with the handling of the vertical plane vibrations.

This topic of transient motions and vibrations could be addressed in a technical engineering sense, which would involve instrumenting the rifle to measure the effects and also modeling the vibrations using finite element analysis computer software. The reader is specifically referred to Chapter #4 of the book, ***Rifle Accuracy Facts*** by the late Harold R. Vaughn, which is available from Precision Shooting, Incorporated, for an enlightening discussion of recoil motion and barrel vibration from an engineering standpoint. In fact, most of the illustrations used in Part I of this article are taken from that book by permission of the publisher. There is no real substitute for this technical approach of making theoretical predictions and making physical measurements that agree within the expected uncertainty for bringing new understanding to an engineering problem. This approach to gaining new understanding of a problem is called the "scientific method."

In contrast, these topics are addressed here in layman's terms as far as possible and from the practical "cut-and-try" approach of gunsmithing and shooting rimfire and centerfire rifles for best accuracy or in benchrest competition. The technical terms that must be introduced to discuss the topic are fully explained and illustrated by example. So, instead of researching the topic of barrel vibrations and adding real knowledge to the field, we intend merely to present some rifle-building techniques that seem to enhance accuracy along with some hand-waving arguments about why we think they work.

Readers wishing to repeat this learning experience for themselves in the most concentrated, efficient manner possible, are encouraged to undertake the converting of bolt action rimfire target rifles into competitive 7.5-pound "Sporter Class" or "10.5-pound Benchrest Class" rifles for 50-yard benchrest competition. One cannot accomplish this without learning a lot about both bench shooting technique and about the control of barrel vibrations. With its relatively slow ignition process and longer barrel dwell time, the rimfire target rifle bullet is extremely sensitive to barrel vibrations that have had plenty of time to develop and to affect the muzzle long before the bullet clears the muzzle crown. Of course, you may also learn that you want to stick with reloadable centerfire rifle chamberings henceforth and forevermore.

Part I of this article contains some general information on mechanical vibrations, some specific information on how rifle barrels vibrate, and an introduction to the mathematics of the Simple Harmonic Motion of the barrel muzzle. A new treatment is presented showing how muzzle vibrations affect the bullet's trajectory to the distant target. The fun stuff in Part II of this article includes a discussion of vibration sources in rifle shooting, how they can be managed in rifle design and construction, and a discussion of barrel tuners and how they work.

A Few Notes on Mechanical Vibrations

Mechanical systems can vibrate in many different ways. A rifle barrel can and will vibrate simultaneously and independently in torsion about its bore, in longitudinal pressure waves (acoustical vibrations), and in lateral (transverse) displacement waves traveling back and forth along the barrel. With its radial symmetry about its bore, the rifle barrel's lateral vibration modes in a vertical plane and in a horizontal plane will tend to be strongly correlated with each other. Furthermore, a single mechanical system can simultaneously and independently vibrate laterally in two or more modes of the same type of vibration. These vibration modes correspond to different standing wave patterns and have different resonant frequencies that may, or may not, be harmonically related. We are only concerned here with vertical plane, transverse displacement waves for reasons that have been mentioned, but these vertical-plane, transverse waves will occur simultaneously in different and independent vibration modes. The mechanical system vibrates as the sum of these independent and simultaneous vibration modes. Transverse waves propagate along the barrel at high speed and at right angles to the displacements.

Whenever a wave encounters a discontinuity in its propagating medium (like, for example, a step change in barrel diameter, an attached mass, or the end of the barrel), some portion of its energy will be reflected and some will generally continue onward with reduced amplitude and with a modified waveform. The reflected wave may be in-phase or phase-reversed, depending on the nature of the reflection. The energy content of a vibrating system at any instant in time is the sum of the kinetic and potential energies of all the parts of the system. Each oscillating element of the system is continually swapping between kinetic and potential energy so that its sum is long-term, fairly stable. Friction losses gradually convert the wave energy into heat energy as the wave dissipates exponentially. The kinetic energy content of a transverse vibrational wave is proportional both to the square of the frequency and to the square of the amplitude of those vibrations.

Mechanical systems are excited into vibration by some type of driving force capable of displacing a portion of the system from its rest position. Fortunately for us, the driving force must contain the resonant frequency of a given vibration mode in order to excite that vibration mode. Unfortunately, however, the kinds of very sharp, short-duration driving forces (impulses) that occur in our rifles contain a wide spectrum of driving frequencies. In fact, the shorter the time duration of the impulse, the broader the frequency spectrum it contains. Much smaller long-duration oscillating driving forces (pumping forces) that match a resonant frequency of a mechanical system can build up quite large vibrations over time.

A mechanical system, like our rifle barrel, “knows” its possible vibration modes and their corresponding frequencies even when it is quiescent. If we could excite the barrel into any arbitrary vibration waveform, it would quickly settle into discrete frequency modes of vibration. Even the very first, smallest responses of the barrel to any driving impulse are already in the discrete vibration modes that we will consider below. Just try to impart arbitrary waveforms into a hand-held fly rod for example, and you will quickly find that only the same discrete modes keep recurring.

The first five of these discrete, vertical-plane, transverse vibration modes which can be excited simultaneously and independently in our rifle barrels are shown schematically in Figure 4-25, taken from Chapter 4 of Harold Vaughn’s book. The waveforms shown are textbook examples for an ideal cantilever beam, and the amplitudes are greatly exaggerated for clarity, but the frequency annotations are typical for a 24 inch hunting rifle barrel. The resonant frequencies for these discrete vibration modes will all shift higher for shorter, stiffer barrels, or downward for longer, slender barrels. Real cantilever beams, certainly including our rifle barrels, always differ to some extent from the ideal vibration modes by not being fixed with perfect rigidity at the left-hand end as shown in these diagrams. Modes 1 (71 hertz) and 2 (445 hertz) do not occur in significant amplitudes because of being difficult to excite in a real barrel and because of very little driving energy being available at these low frequencies. Mode 3 (1246 hertz) does occur in our rifle barrels and is the most significant vibration type discussed in the rest of this article. Modes 4 (2429 hertz), 5 (4036 hertz), and higher order modes do occur in our rifles, but the amplitudes typically decrease by an order of magnitude, or more, for each mode number above Mode 3. Mode 6 and higher vibration amplitudes would normally measure as a few millionths of an inch and are not usually of concern while lower numbered vibration modes are effectively “masking” them.

Vibration Effects in a Standard Rifle Barrel

The standard rifle barrel has its crown at the muzzle and has no muzzle attachment such as a muzzle brake or tuning device. The effects of these attachments are discussed in the last section of Part II of this article. The vertical plane, transverse wave vibrations of the barrel produce significant and predictable effects in bullet impact locations on the target downrange. Conversely, since we do not see similar horizontal, elliptical or diagonal effects in groups fired with a well-made rifle, we reckon that the other possible types of barrel vibrations are not occurring to a significant degree.

The muzzle end of an ordinary rifle barrel, like any other point of the vibrating barrel, moves up and down in a fashion called "Simple Harmonic Motion" in physics. This type of motion occurs whenever an object has a central, "neutral," or rest position and is allowed to move freely in one dimension, subject only to a centripetal "restoring" force that is directly proportional to the amount of that object's displacement (distance) from its central rest position. The familiar clock pendulum exhibits approximately this kind of motion if it swings only through a small arc. In the case of our rifle barrel the central rest position of the muzzle is its position and orientation with the rifle in its firing position (including "gravity droop" of the barrel) just prior to firing. The free muzzle end of the barrel moves up and down with a sinusoidal motion in the vertical plane when the barrel system is excited into transverse "standing wave" vibration by vertical plane impulses arising during the firing process. The recoil reaction force acting on the rear face of the recoil lug at the front of the receiver directly produces an upward bending moment on the rear of the barrel. A standing wave is a stable type of transverse vibration that appears to be stationary along an extended linear object such as a guitar string or a rifle barrel. It should be pointed out here that these standing waves along a rifle barrel are **not** actually true sinusoidal waveforms, but are more or less distorted sine waves. This fact does not conflict in any way with the statement that the individual elements of the barrel are moving in true Simple Harmonic Motion (SHM). As time marches onward, the vertical position of the muzzle itself, plotted against time, describes the well-known "sine wave" pattern of motion, which is exactly the mathematical solution to the "equation of motion" we just verbally described above for SHM. The maximum displacement above or below the rest position is called the "amplitude" of the vibration, and may be about 0.001-inch, or less, for the muzzle of a typical rifle barrel. From the handbooks, the velocity of propagation for these transverse vibrations in steel rifle barrels is about 3150 meters per second (or about 10,335 Feet per Second).

The muzzle comes to a momentary stop at each maximum displacement point while it is reversing its direction. The muzzle's rate of motion, or speed in inches per second, describes another sine wave with its velocity peaks (maximum speeds of transverse motion) occurring at the same times as the zero crossings (original rest locations) in the position sine wave. The sum of the positional (launch angle) effect and the vertical velocity (velocity kick) effect at the time of the bullet's exit from the barrel crown produces the change in bullet impact point that we see on a distant target while tuning for best rifle/ammo accuracy. As soon as the bullet has exited the barrel muzzle and the subsequent blast cloud, the bullet becomes a rapidly spinning "free body" that follows an unguided "ballistic trajectory" to the target subject only to aerodynamic forces and moments (chiefly atmospheric drag force) and to the acceleration of gravity. Of course, not all of the aerodynamic forces may be knowable in the real world of an outdoor match, and the spinning bullet reacts similarly to the ways in which a toy gyroscope would react to the turning moments produced by these aerodynamic forces. Aside from its spin characteristics and the above-mentioned drag and gravitational influences, the bullet's initial "state vector" [including its position, orientation, and velocity vectors at an instant in time after the bullet has exited the muzzle blast cloud] completely determines the bullet's future trajectory and eventual target impact point. Furthermore, barrel vibrations can affect the bullet's state vector **only** through the launch angle and velocity effects mentioned above. Accelerations and higher order effects all go to zero immediately after the bullet exits the barrel crown and clears the muzzle blast cloud.

Therefore, we can see that the effects of vertical plane barrel vibrations on bullet impact point on the target can be analyzed (separated) into a displacement effect (that becomes a launch angle effect) and a velocity kick effect at the instant of bullet exit from the barrel crown. These two effects will be treated separately and brought into the same units so that they can then be recombined. This approach to studying a problem is called "the analytical method."

The Position Function

Since most ordinary hunting rifle barrels predominantly vibrate in Mode 3 at about 1250-hertz as shown both in Figure 4-25 and in Figure 4-28 in Harold Vaughn's book, ***Rifle Accuracy Facts***, we will use this vibration mode here in our explanatory example. A barrel vibrating in Mode 3 will have its front node (stationary point) in the standing wave barrel vibration pattern about 10% of the barrel length back from the muzzle end, or about 2.4-inches back for our example 24-inch, normally tapered, hunting rifle barrel. This Mode 3 vibrational displacement of the muzzle (of usually less than 0.001-inch from the central rest position) effectively causes the free muzzle end of the barrel to ***pivot*** about the foremost barrel node and, therefore, to cause a variation in the pointing direction of the muzzle. This pointing error significantly changes the launch trajectory of the bullet depending on just when in the vibration cycle the bullet exits the muzzle. If, for example, the outer 2.4-inches of our barrel is pivoting up and down by 0.0005-inch each way at its distal end, this would mean bullets will be impacting at up to 0.716 minutes of angle (MOA) above and below the aim point at ***any*** given target range due to this launch angle effect alone. This maximum impact error distance is figured mathematically by:

$$\text{ERR}=(60\ \text{MOA/Degree})*(180\ \text{Degrees/Pi Radians})*\text{ArcTan}(.0005/2.4)$$

$$=(3437.75\ \text{MOA/Radian}) * \{(.0005\ \text{inch}/2.4\ \text{inches})\ \text{Radians}\}$$

$$=0.7162\ \text{MOA}$$

The factor of 60 just converts degrees of angle into minutes of angle (MOA). A minute of angle is one sixtieth of a degree of angle and happens to subtend (or cover) 1.0472 inches at a distance of 100-yards. Actually, for this situation, we can use the "small angle approximation" instead of the inverse-tangent function without loss of significant accuracy. This approximation is that for small angular arguments (x), the Tan(x) and Sin(x) are about the same as x ***in radians***. There are 2*Pi radians per full circle of 360-degrees. The mathematical constant Pi is 3.141593… and is the ratio of the circumference of a circle to its diameter. Note that a full circle contains 6283.185… milliradians (2*Pi*1000). The milliradian is the inspiration for the "mil" of mil-dot ranging fame. In fact, when we define the "mil" used in range estimating to be the small angle subtended by a unit-dimension object as seen from a distance of 1000 units, we are using ***both*** the “small angle approximation” and the true “milliradian” defined above.

Some experience in barrel tuning indicates that the size of this angular deviation probably runs from about 0.1 MOA for stiff target barrels up to about 2.0 MOA on each side of the neutral pointing direction for lighter hunting rifle or for longer antique military rifle barrels. For many factory rifles the frequency of this Mode 3 standing wave barrel vibration will be around 1250 hertz (cycles per second). This frequency happens to be about the highest excitation (vibration driving) frequency produced by the chamber pressure impulse that results from burning the powder in most centerfire cartridges. This chamber pressure impulse is what accelerates the bullet down the bore and that acceleration, in turn, causes the barreled action to recoil rearward. The rifle stock then exerts a recoil reaction force upon the recoil lug that, in turn, exerts an upward bending moment directly on the barrel shoulder in most rifle designs. All of these actions occur with the timing as shown in Figure 2-16 of ***Rifle Accuracy Facts*** for typical Chamber Pressure versus Time curves. The pressure impulse often has a half-period pulse width of about 0.400 milliseconds for its main peak. All this is to show how our assumed Mode 3 resonant frequency of 1250 hertz (with a period of 0.800 milliseconds) is likely to be excited into vibrating during firing. Except perhaps for black powder cartridges, most target bullets spend between about 1.0 msec and 3.0 msec in the barrel after they start moving forward, and the bearing diameter portion of the bullet clears the muzzle during the last approximately 1% of that barrel dwell time.

After the bullet exits the muzzle crown, a jet of high-pressure gasses follows behind it and quickly passes it up. These hot gasses exit the barrel at 6000 FPS, more or less, depending on remaining gas pressure and temperature and on bore diameter and barrel length. This rush of hot gasses continues to accelerate the bullet for a few inches and eventually forms a blast cloud surrounding the projectile. This buffeting may destabilize some bullets, but there is no evidence that any redirecting of the muzzle blast gasses by the movement of the barrel muzzle has any systematic steering effect on the bullet. After the bullet emerges from the blast cloud, it settles down into spin-stabilized aerodynamic flight to the target.

We can now write the expression for the position function of the barrel muzzle as:

$$Pos(t) = A * Sin(W * t)$$

where A = Amplitude = 0.0005 inches (in this example)

W = Circular Frequency = 2 * Pi * 1250 radians/second.

As discussed above, this variation in muzzle position at the time of bullet exit can be viewed as a variation in the launch angle of the bullet. Assuming horizontal firing, the effect at the target of the variation in muzzle pointing can be expressed as:

$$EP(t) = (3437.75 \text{ MOA/Radian}) * ArcTan\{(.0005 \text{ inch})/2.4 \text{ inches})\} * Sin(W * t)$$

Or, once again using the small angle approximation:

$$EP(t) = 0.7162 \text{ MOA} * Sin(W * t).$$

This expression for the "launch angle effect" derived from the muzzle position function for this hypothetical barrel vibration will be combined with the velocity function effect developed below.

The Velocity Function

If the muzzle of our example rifle barrel is vibrating up and down in Simple Harmonic Motion through a total distance of about 0.001-inch (twice the displacement amplitude) in cycles of 0.800 msec in duration (as mentioned above), the amplitude of its velocity function would (mathematically) have to be 3.937 inches per second (ips). We know this because the velocity function **must** be given by the **time derivative** of the position function (developed above). If the crown of the barrel is located at the muzzle, the bullet will be given a small, cross-track velocity kick determined by this velocity function evaluated at the time (t) of bullet exit. To evaluate the effect of this incremental vertical velocity kick on the bullet impact at the target, we can simply multiply it by the bullet's time-of-flight to the target. This evaluation approach implicitly assumes (not unreasonably) that the initial velocity kick stays constant over the bullet's rather brief flight time to the target. The time-of-flight for many target bullets (those with muzzle velocities just over 3000 FPS) to reach 100-yards is about 100 msec. So, the velocity function alone would have an amplitude of 0.3937-inches at 100-yards (or 0.3750 MOA) in our assumed-typical example. A more concise method of evaluating the angular effect (in MOA) of a transverse vertical "kick velocity" is to multiply 60 times the Arc-Tangent of the quantity, kick velocity divided by the average bullet speed to the target. The two velocities need to be in the same units, and the result of the Arc-Tangent function needs to be expressed in degrees.

Note that the "average bullet speed to the target" term that we slipped in on the innocent reader indirectly implies a particular **range** to the target since real bullets are constantly slowing while in flight. This slight range dependence grows only to 0.497 MOA at 1000-yards for the example bullet being discussed. [By the way, if we had gone off into a "Conservation of Angular Momentum" argument to evaluate the effects of this velocity function, we would have just taken longer to arrive at this same place.]

The equations expressing what was just verbally described are:

$$\mathrm{Vel}(t) = d/dt\,[\mathrm{Pos}(t)] = d/dt\,[A * \mathrm{Sin}(W * t)]$$

$$= A * W * \mathrm{Cos}(W * t) = A * W * \mathrm{Sin}(W * t + \mathrm{Pi}/2)$$

And, the effect at the target for our example of this velocity kick is given by:

$$\mathrm{EV}(t) = (3437.75\ \mathrm{MOA/Radian})*\mathrm{ArcTan}\{(A*W)/(12*3000)\}*\ \mathrm{Sin}(W*t + \mathrm{Pi}/2)$$

where

> (12 inches/foot)*(3000 feet/second) = Average bullet velocity (assumed for this example) converted into inches per second to a range of 100 yards (the target range in this example).

Using the small angle approximation once more, we have:

$$\mathrm{EV}(t) = 0.3750\ \mathrm{MOA} * \mathrm{Sin}(W * t + \mathrm{Pi}/2).$$

Note how this velocity function amplitude magically agrees with the “kick velocity” multiplied by the bullet’s time-of-flight to the target. This is called “prestidigitation.”

The Combined Function

Having been analyzed separately and with the results put into the same units (MOA), these two vibrational effects can now be recombined. They always occur together, even if the velocity effect is much reduced via the use of a barrel tuner, as discussed in Part II of this article. We have to **sum** two sinusoidal functions having the same periods, but differing amplitudes, and always being 90-degrees (or Pi/2 radians) out of phase with each other. Saying the two waves are “90-degrees out of phase” just means that one sine wave is shifted by a time delay (0.200 msec in our example) corresponding to one quarter of a full cycle (360 degrees) with respect to the other. The concept of “phase shift” implies that we are dealing with waves of the same frequency. Oddly enough, the arithmetic (point-by-point) summing of two sine waves of the same frequency always produces a third sine wave of that same frequency, but with an amplitude and phase shift to be computed. The sine wave plots shown in Graph 1 illustrate the summing of these two functions for our typical example case, which combines the two effects into the form in which we would actually encounter them in ammunition-versus-barrel tuning.

For this example, we need to sum the two expressions below in order to re-combine the muzzle position and velocity effects in terms of bullet impact point at the target:

$$EP(t) = 0.716 \text{ MOA} * \text{Sin}(W * t)$$

$$EV(t) = 0.375 \text{ MOA} * \text{Sin}(W * t + \text{Pi}/2)$$

Now, these two sine waves can be summed mathematically as “phasors” since they differ only in amplitude and phase, but have the same frequencies. This is an Electrical Engineering concept useful in dealing with AC power distribution and with AC machinery and is similar to vector addition in Physics, but it has nothing to do with the Star Trek TV series nor with futuristic weapons that can be set to “Stun.” The polar plot shown in Graph 2 depicts the summing of these two phasors for this example such that:

$$\text{Sum}(t) = 0.8084 \text{ MOA} * \text{Sin}(W * t + 0.4825 \text{ radians})$$

The Sum(t) function is what we observe on targets at 100-yards (in this example) as we shoot 5-shot groups using small increments in powder charge between the groups (but correcting for variations in bullet drop due to changes in average muzzle velocity for each group) so as to slowly vary the time of bullet exit with respect to the muzzle vibration cycle. This muzzle vibration cycle is assumed to occur the same way for each individual shot. The calculated phase angle of 0.4825 radians corresponds to 27.6 degrees, which is closer to the launch angle (muzzle position) effect than to the velocity kick effect. In this example, we would see the group centers sweep between plus 0.8084 MOA and minus 0.8084 MOA (corrected for expected bullet drop variations) separated by 0.400 msec in bullet exit times (one half a vibration cycle at 1250 hertz or cycles per second). We also expect to find the **smaller** group sizes at or near these extremes of the Sum(t) function. The large size of the change in muzzle velocity corresponding to a change of 0.400 msec in barrel dwell time is the reason that in tuning the ammo to the barrel the best accuracy usually occurs at “too slow” and “too fast” muzzle velocities.

To get away from our example and consider the case of any standard barrel, the sum function can be expressed more generally as:

Sum(t) = Mag * Sin(W *t + Phi)

where Mag = K * SQRT{(A/Ndis)**2 + (A * W/Vbar)**2}

Phi = ArcTan{W * Ndis/Vbar}

K = 60 * 180/Pi = 3437.75 MOA/radian

Ndis = Nodal distance from muzzle in inches

Vbar = Average bullet velocity to target in inches per second

W = 2*Pi * Frequency(in hertz) = Circular Frequency (rad/sec)

This expression for the Combined Function Sum(t) holds separately for any transverse barrel vibration mode of Mode 2 or higher. The expression for the Phase Angle Phi shows that as the dominant frequency W increases, Phi approaches Pi/2 radians (or 90 degrees). Moreover, this shows why for stiffer barrels with higher vibrational frequencies, the velocity kick is the dominant vibration effect moving the bullets on the target. This is probably why Harold Vaughn used the velocity kick effect in explaining barrel vibrations throughout his book. Conversely, for a long thin barrel that vibrates with a lower resonant frequency (about 500 hertz), the variation in launch angle caused by muzzle vibrations is the dominant effect. Hence, the early 1900's British explanations of "compensation" in launch angle terms for their long-range SMLE rifles in .303 caliber. Actually, both effects were present together in each of these extreme cases.

Figure 4-41 shows a graph of group height versus muzzle velocity plotted with bullet drop variations removed for an accurate 6mm PPC heavy varmint benchrest competition rifle. This barrel has a residual 6.7Khz barrel vibration, and best accuracy is obtained either at just below 3100 FPS or at just over 3300 FPS. The magnitude of the combined vibration functions is 0.2865 MOA as shown (0.300 inches). In this case, the position function is only 0.1081 MOA while the dominant velocity function computes to 0.2653 MOA, and the Sum(t) function is phase shifted by 67.8 degrees from the position function. The amplitude of the barrel vibration producing these effects is 66 micro-inches at the muzzle of this benchrest barrel.

Figure 4-39 shows another plot of group impact height, with gravity drop corrected, for a heavy barreled 6mm BR rail gun fired at about as wide a range of muzzle velocities as is possible. It shows a 9500 hertz residual barrel vibration effect attributable to its Sum(t) function. The smaller groups were fired at a maximum point corresponding to 27.0 grains of H322 powder and at a minimum point corresponding to 28.0 grains of H322. Back-figuring the Pos(t) and Vel(t) functions for this case indicates the Sum(t) function is at 71.5-degrees from the Pos(t) function which has a magnitude of only 0.04 MOA, while the Vel(t) function has a magnitude of 0.11 MOA for this accurate rail gun. The vibration amplitude for this barrel muzzle is only 20 micro-inches at this high frequency.

These few examples show that as the frequency of the residual vibration effect increases as in heavier barreled benchrest rifles, the velocity function tends to increasingly dominate the effects seen at the target. These examples also show that what was said earlier about the very-small-amplitude, high-frequency vibration modes not being significant, really applies only when these effects are being masked by much larger, lower-frequency barrel vibration modes.

End of *Part I*