The Bullet's Flight through Crosswinds

Part II

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The New Formulation

We can take the ***standard analytical formulation*** of bullet movements in flight to be essentially those formulas presented by Robert L. McCoy in his book, *Modern Exterior Ballistics*, Schiffer, 1999—particularly as outlined in Chapters 10 through 13 of that book. Further, these formulations are representative of those used in the modern era by the US Army's Ballistics Research Laboratory (BRL) in their work at Aberdeen Proving Ground in Maryland. These analytic formulations start with an *a priori* assumption that ***no crosswinds*** affect the bullet's flight. These formulations include accurate computations of the bullet's no-wind trajectory, fast- and slow-mode spin-axis motions, and the radius of its epicyclic swerve motion. Much emphasis is placed upon the curlicue motions of the bullet's spin axis direction plotted against the pitch-versus-yaw "wind axes," as if this figure completely explains bullet motions in flight. The particular details of the spin-stabilized bullet's motions that are discussed are not integrated into a single coherent formulation. For example, basic crosswind drift is calculated separately by Didion's method of 1859 and then just tacked on. Also, the bullet's "coning motion" is *not introduced* to tie together the bullet's spin-axis motions with the helical "epicyclic swerve" motion of its center of mass. Does the bullet fly around in its steady helical spiral about its mean trajectory with its nose angled *outward* at its angle of attack, or *inward* toward its mean path, or somewhere in between? The correct answer, *inward*, is not discussed.

The "excessive lift problem," mentioned at the end of *Part I* of this article in the June 2008 issue of *Precision Shooting*, is used as an illustrative example of what can happen in the absence of an *integrated formulation* for the motions of a spin-stabilized bullet. The ***new formulation*** presented herein incorporates crosswind handling by tying the details of the bullet's motions in flight into its actual "coning motion." Physically, this coning motion can be defined as an ***isotropic harmonic oscillation*** driven by the ***aerodynamic lift force*** attributable to an angle of attack equal to the (half) ***cone angle***. The axis of this coning motion always quickly aligns itself directly ***into the apparent wind*** in both pitch and yaw angular coordinates. The aerodynamic overturning moment due to a crosswind would turn the nose of the non-coning bullet ***away from the wind***. Instead, the coning bullet accomplishes its alignment feat by ***selectively enlarging its cone angle*** so that, while the spin-axis of the bullet itself does indeed point farther from the wind, its ***average pointing direction*** (i.e., the direction of its ***cone axis***) points directly into the wind. It is this alignment of the cone axis into the wind that eliminates the "excessive lift problem" mentioned earlier. The ***average lift force*** on the stable, coning bullet ***goes to zero***. Unlike Bob McCoy and BRL, we do not have to concern ourselves here with extending our new formulation to cover missiles, bombs, and non-spinning projectiles.

Bullet Coning Motion

For this example projectile (a 30 caliber Sierra 168 grain International bullet), the data reported by BRL shows the slow-mode motion of the bullet's spin axis, or its "coning motion," to be *undamped*, so this cone angle slowly *increases* throughout the flight. We are cautioned by Bryan Litz, a working aerodynamicist, ballistician and long-range marksman who is familiar to readers of *Precision Shooting* magazine, that this example bullet, and its direct descendant, the 168 grain Sierra MatchKing, are *unusual, if not practically unique,* in exhibiting this undamped slow-mode coning motion. We should not assume that the undamped coning motion of this example type of bullet is *typical* of most other match bullets, which normally have the *dynamic stability* to damp out this coning motion soon after being initiated by a flight disturbance. Depending upon the bullet's dynamic stability values, the size of its slow-mode coning motion (yaw angle) may either damp down or increase (or accumulate) as its flight progresses. We study this particular bullet both because we have access to its flight data from BRL and because it is an interesting and instructive bullet to analyze.

Table 1 shows typical values for our example 168 grain bullet of these cone angles increasing as the flight progresses, and it also shows that the time rates of its coning motion are continually slowing throughout the bullet's flight, starting at 62.5 hertz at 2600 FPS and slowing to 16.6 hertz at 900 yards downrange and 1160 FPS bullet speed. The coning of a spinning bullet consists mostly of the back end of the bullet swinging around in a clockwise circle while the nose stays close to the trajectory. This coning motion was illustrated in Figure 7 in *Part I* of this article and is repeated herein. The path described by the bullet's spin axis is a forward-pointing right-circular cone about the path of its mean trajectory as would be seen by an observer flying alongside the bullet. I do not know who coined the expression "coning motion," or when the term came to be used by some ballisticians to describe the bullet's slow-mode, gyroscopic-precession-like motion, but it is particularly apt, as it accurately describes both the motions of the bullet and those of its pointing direction.

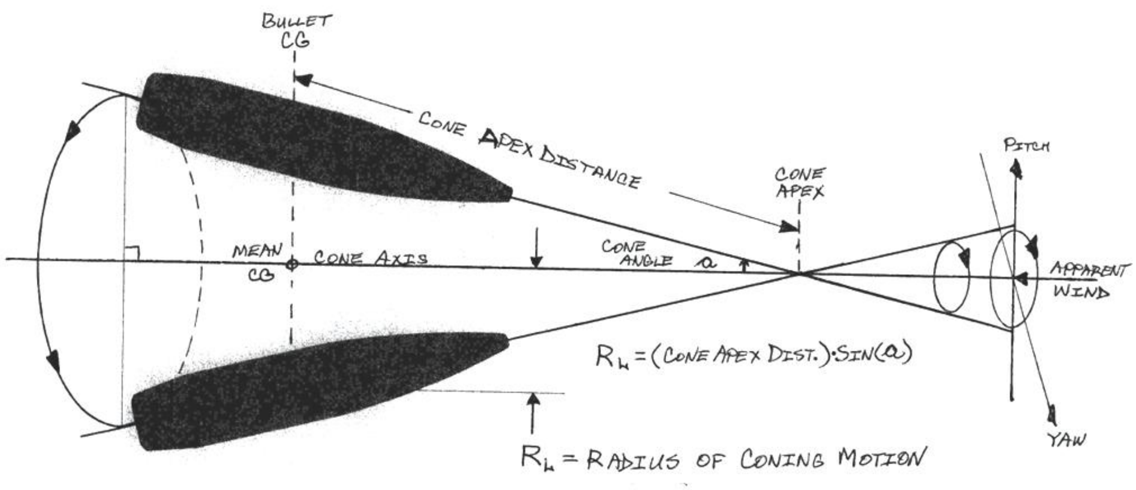


FIGURE 7. BULLET CONING MOTION

In the BRL formulation, the bullet's slow-mode rate depends primarily upon physical constants of the bullet, upon the twist rate of the barrel, and upon the density of the atmosphere. The bullet's coning rate depends directly upon its spin rate as the bullet slows both its forward velocity and its spin rate throughout the flight, and depends only indirectly upon the bullet's Mach number (speed through the air) by way of its dependence upon the overturning moment coefficient **CM(a)** that, in turn, varies with Mach number. As is proper for a harmonic oscillator, the bullet's coning rate *does not* depend directly upon the total angle of attack, the cone angle **a** (i.e., on the *amplitude* of the oscillation), other than as a small corrective term for **CM(a)**. The bullet does not even have to be coning or oscillating at all to have both its fast-mode and slow-mode rates of motion defined. While the overturning moment and the spin of the bullet are both probably necessary for establishing and maintaining the coning motion of the bullet, I have yet to work out any of these physical connections. But I expect that the spin stabilization of the bullet will turn out to be necessary for the "circularization" of the coning motion, for example. While the standard BRL formulation matches the observed ballistics of bullets, bombs and missiles quite well, I am not sure that we really know *exactly why* it does so. And I suspect that "coning motion" was omitted from Bob McCoy's works on the standard formulation precisely because it is an artifact of the bullet's spin-stabilization with no counterpart in the motions of fin-stabilized missiles.

The bullet's time-rates of coning motion are shown in Table 1 at selected points during its flight. This stable, circular coning motion is a rotationally symmetric, ***isotropic harmonic oscillation*** of the bullet's center of mass about a neutral axis defined by the direction of the apparent wind. This oscillating motion is driven by the centripetal-acting aerodynamic lift force acting at the CG of the bullet, that is itself proportional to the orbital radius **RL** of the coning motion (through its dependence on the sine of the cone angle **a**). As shown in Table 1, the amount of ***lift force*** attributable to an angle of attack equal to the current coning angle **a** at any point in the bullet's flight, turns out to be ***just exactly the centripetal force needed to drive the known mass of the bullet inward, toward the axis of the cone, so that the bullet's center of mass will orbit at the expected radial distance about the axis and at the expected cyclic rate of the coning motion*.** I discovered this relationship for myself by formulating a cross-check expression for the circular orbital radius **RS** of a *mass* equivalent to our 168 grain bullet subjected only to the tabulated *lift force* attributable to an angle of attack **a** and rotating at the tabulated *coning rate,* at 100-yard range increments. I found that my calculated values for **RS** at each range agreed within less than one percent with the tabulated values of **RL**, the radius of the bullet's helical path around the mean trajectory, as formulated by BRL. [I have since discovered this same relationship for calculating the orbital radius of the bullet's "corkscrew motion" in Harold R. Vaughn's wonderful book, *Rifle Accuracy Facts* (published by *Precision Shooting* in 1998), on page 192 in Chapter10 on External Ballistics.]

Table 1. Studies of .308 Sierra 168gr International Bullet in ICAO Atmosphere

Range	yards	Muzzle	100	200	400	600	900	1000	Yards
Time of Flight, t	seconds	0.0000	0.1200	0.2501	0.5461	0.9033	1.5899	1.8613	seconds
Bullet Velocity, V	ft/sec	2600.0	2403.2	2214.9	1852.0	1524.4	1148.2	1069.6	ft/sec
Mach Number	(A=1116.45 ft/sec)	2.329	2.153	1.984	1.659	1.365	1.028	0.958	
Spin Rate, p	cycles/second	2600	2558	2516	2428	2337	2192	2142	hertz
Spin/cal (pd/V)	rad/caliber	0.1613	0.1745	0.1863	0.2153	0.2521	0.3147	0.3305	rad/cal
Cone Angle (a)	degrees	1.018	1.54	1.7	2	2.3	5	5	degrees
Del=Sin(Cone Angle, a)		0.0178	0.0269	0.0297	0.0349	0.0401	0.0872	0.0872	
Aeroballistic Coefficients (from BRL Data):									
Coef of Drag, CD(0)		0.331	0.34	0.353	0.374	0.412	0.446	0.41	
Drag Corr, CD(2)		4.96	5.58	6.1	7.2	7.18	3.08	2.9	
CD=CD(0)+Del^2*CD(2)		0.3326	0.3440	0.3584	0.3828	0.4236	0.4694	0.4320	
Coef of Mom, CM(a0)		2.63	2.71	2.78	2.95	3.07	3.2	3.42	
Mom Corr, CM(a2)		-4.4	-4.4	-4.4	-4.4	-4.4	-4.35	-4.3	
CM(a)=CM(a0)+Del^2*CM(a2)		2.6286	2.7068	2.7761	2.9446	3.0629	3.1670	3.3873	
Coef of Lift, CL(a)		2.75	2.65	2.58	2.36	2.11	1.46	1.31	
Spin Damp, CL(p)		-0.00708	-0.00735	-0.00750	-0.00820	-0.00889	-0.00990	-0.01030	
Calculated Forces and Moments:									
Dynamic Force, DYN	pounds	4.16	3.55	3.02	2.11	1.43	0.81	0.70	pounds
Drag Force=DYN*CD	pounds	1.3830	1.2223	1.0815	0.8076	0.6055	0.3807	0.3041	pounds
Lift Force=DYN*CL*Del	pounds	0.2032	0.2530	0.2310	0.1738	0.1211	0.1032	0.0804	pounds
F = SQRT(D*D+L*L)	pounds	1.3978	1.2482	1.1059	0.8261	0.6175	0.3944	0.3145	Pounds
L/D Ratio		0.147	0.207	0.214	0.215	0.200	0.271	0.264	
Angle b=ARCTAN(L/D)	degrees	8.36	11.70	12.06	12.14	11.31	15.17	14.80	degrees
R = CP-CG = CM(a)/(CD+CL(a))	calibers	0.853	0.904	0.945	1.074	1.209	1.641	1.944	calibers
Moment=(d/12)*DYN*Del*CM(a)	lbs-ft	0.004985	0.006634	0.006379	0.005565	0.00451	0.005746	0.005333	lbs-ft
Computed Tri-Cyclic Motions:									
P = (Ix/Iy)(pd/V)	radians/caliber	0.021645	0.023417	0.024998	0.028891	0.033835	0.042242	0.044363	rad/cal
M = (ky^-2)CM*a	(rad/cal)^2	6.89E-05	7.1E-05	7.28E-05	7.73E-05	8.04E-05	8.38E-05	8.96E-05	(rad/cal)^2
Fast Mode Oscillation Rate	(cycles/sec)	286.4	295.6	297.1	297.5	295.5	285.9	280.2	cycles/sec
Slow Mode Coning Rate (CR)	(cycles/sec)	62.51	53.32	46.23	34.25	24.31	14.86	14.07	cycles/sec
Ratio of Rates	(Fast/Slow)	4.58	5.54	6.43	8.69	12.15	19.24	19.92	
RL=ky^2*(CL/CM)*Ratio*Del	inches	0.021	0.036	0.044	0.060	0.084	0.192	0.167	inches
Apex Dist=RL/(d*Del)	calibers	3.87	4.38	4.82	5.62	6.76	7.16	6.22	calibers
Calculated Cross-Check Data:									
Torque=(d/12)*R*F*Sin(a+b)	lbs-ft	0.004984	0.006632	0.006377	0.005563	0.004507	0.005729	0.005318	lbs-ft
[Matches "Moment" data above.]									
RS=(3*g/(Pi^2*w))*L/CR^2	inches	0.021	0.036	0.044	0.060	0.083	0.190	0.165	inches
100*(RL/RS-1)=Percent Delta		0.05	0.12	0.14	0.18	0.23	1.04	0.96	percent
[Matches "RL" Orbit Radius data above.]									
Gyro Nutation Rate=(Ix/Iy)p/2Pi	hertz	348.96	343.33	337.69	325.86	313.65	294.24	287.54	hertz
[Do NOT Match Fast Mode Oscillation Rates above.]									
Gyro Prec Rate=Mom/(2Pi*p*Ix)	hertz	0.91	1.23	1.21	1.09	0.92	1.25	1.18	hertz
[Do NOT Match Slow Mode Coning Rates above.]									

We depend upon the coning motion of the bullet to disperse this rather large lift force due to a bullet's angle of attack **a** without appreciably altering the bullet's trajectory, but, at the same time, we now need to revisit our explanation of how the coning bullet produces the slight downward lift that would explain the observed secondary vertical effect of firing through a purely horizontal, left-to-right crosswind, as discussed in our previous article. In light of this new understanding of the bullet's coning motion, we also need a new explanation of how the coning bullet produces a small rightward "yaw of repose" (if indeed that tiny effect is even real), or at least we need an explanation of how the coning bullet produces the well documented, slow rightward drift of the right-hand-twist bullet at long ranges. And finally, we need to explain how this coning bullet is able to "arc over" to track the evermore downward curvature of the trajectory so that the bullet impacts the target at least approximately point forward, as it is observed to do in flat firing on target ranges.

Coning Motion Is Necessary

The rapid **coning motion** of the spinning bullet in flight is **absolutely necessary** in order effectively *to use up and cancel out* this overly-large aerodynamic lift force so that we can safely avoid suffering wind sensitivities about an order of magnitude larger than those we actually experience. But, in fact, since the ***lift force itself*** arising from any ambient crosswind ***causes and initiates the bullet-coning motion***, the extra wind sensitivity mentioned here ***cannot occur in the absence of coning***. This revolving lift force effectively cancels itself out (as far as producing wind drift) after the bullet achieves a rapid, circular, "coning" motion about its trajectory at a cyclic rate of 62.5 revolutions per second at a forward speed of 2600 FPS for this particular type of bullet and barrel twist rate.

Let me quote Dr. Mann (from page 253 of my 1942 edition of *The Bullet's Flight*):

> *"Bearing in mind the fact that the instant a bullet flying in the air begins to tip it also begins to gyrate,…therefore it will be readily comprehended that a tipping bullet must gyrate in order to reach its target."*

"Tipping" refers to flying at a non-zero angle of attack, as when encountering a crosswind, and "begins to gyrate" means to commence coning motion in my terminology. The ellipsis in the middle of his paragraph-long sentence omits a description of the *excessive lift problem* similar to my discussion above, but couched in the antique terminology of the nineteenth century and not specifically tied to a crosswind as the precipitating cause. Keep in mind that Dr. Mann was working contemporaneously with the Wright brothers as they were developing their theory of flight. At that time in the history of ballistic developments, the recent switch to smokeless powder had just made possible the widespread use of smaller caliber, higher velocity, longer and heavier-for-caliber rifle bullets. Dr. Mann must have noticed that these new-fangled bullets flew quite differently than did the older patched round balls, minie balls or paper-patched lead slugs. Here, today, we are still studying an example of the then-newly-developed jacketed, Spitzer-type, pointed bullet.

Coning Is Not Gyroscopic Precession

If this bullet's time rate of coning, as shown in Table 1, were actually a gyroscopic precession driven by the overturning moment due to its initial **1.02-degree** cone angle (in Robert L. McCoy's example flight of this bullet and his matching 6-DOF simulation at BRL), its actual initial precession rate would calculate to **0.91 cycles per second**. This gyroscopically precessing bullet would then be flying in a *much larger in diameter* and *much slower rate* helix about its trajectory. It probably would not even complete one cycle in 600 yards. And this type of bullet behavior in flight is simply *not* what has been reported with this, or with any other, bullet. [For years I thought this slower precession rate was exactly what I *was* seeing whenever I observed the track of bullets in flight. But, based on the BRL data, I must assume that I had been seeing some type of optical distortion or illusion. Anyway, I was really surprised to learn that the bullet's slow-mode rate of motion was over 60 hertz. By the way, this initial **0.91-hertz** time rate of gyroscopic precession was calculated by dividing the overturning torque (**0.00498 pounds-feet** just after launch) by the bullet's angular momentum (**0.0008702 pounds-**

feet-seconds). I then divided this angular precession rate (in radians per second) by **two pi** radians per revolution to convert the precession rate into cycles per second (or hertz).]

The bullet's observed initial coning rate of **62.5 hertz** does not match the calculated **0.91 hertz** time rate of gyroscopic precession, nor, as shown in Table 1, do the calculated time rates of gyroscopic nutation agree particularly well with the bullet's observed rates of fast-mode "oscillations," to borrow Dr. Franklin W. Mann's 1909 terminology. ***The coning motion is not simple gyroscopic precession.*** Perhaps this is why Dr. Mann consistently referred to this bullet motion as "gyration," even though he was well aware of gyroscopic motions, and why BRL simply labels this same phenomenon as the "slow-mode" motion in the modern tri-cyclic theory. And instead of "nutation," Dr. Mann consistently referred to the "fast-mode" motion of the bullet as "oscillation." By the way, from many years of carefully observing his experiments, Dr. Mann knew and reported (on page 267 of my 1942 edition of *The Bullet's Flight*) that *increasing the twist rate of the barrel increased the bullet's oscillation rates and decreased its gyration rates*, which observation agrees with the BRL formulations used in calculating the values in Table 1.

Description of Coning Motion

The slow mode motion of the right-hand spinning bullet takes the form called "coning motion," as shown in Figure 7, because the bullet moves as if it were rolling around clockwise inside of a small conical funnel pointed generally forward along the trajectory. At small cone angles (under one degree), the point of the bullet practically follows along the 3-DOF point-mass trajectory path—or at least the bullet nose more closely follows the trajectory than does its afterbody or its center of mass. Let me again quote Dr. Franklin W. Mann on this subject from page 273 of *The Bullet's Flight*:

> *"…when the bullet is at the top of its spiral, it is tipping directly downward; when going downward the fastest on the left side of its spiral, being left twist, its point stands toward 3 o'clock, where it would naturally be expected its deflection to be most rapid to right instead of downward as found."*

And, a few sentences later:

> *"…its point seems to be 90 degrees in advance."*

Given that he was using left-hand-twist Pope barrels, instead of our now standard right-hand-twist direction, this seems to be a pretty clear description of a left-twist version of our lift-driven "coning motion." He is describing a counter-clockwise coning motion of the left-hand spinning bullet as seen from the shooter's perspective.

Lift Force Drives the Bullet Coning Motion

Throughout the bullet's flight, its aerodynamic lift force provides <u>exactly</u> the *centripetal force* needed to cause the bullet's center of mass to *orbit* around the mean path of its (3-DOF) trajectory in a circular orbit with the radius and velocity values at each point along the trajectory matching those defined in studies of this bullet published by BRL. In fact, the agreement is so uncanny that I suspect that perhaps BRL derived the relationship for finding **RL** for a given bullet by using the same basic orbital mechanics that I used in my cross-check calculation of the orbital radius values **RS**, computed from the bullet's mass and the tabulated size of its lift force and time-rate of coning motion at each range, as shown in Table 1. Even if this suspicion proves correct, my argument here still holds because these values of **RL** are reported to agree with *observations* of the bullet in flight. [These very small radius values also generally agree with those reported by Dr. Mann for his bullets, as well.] These two different sets of radius values, **RL** and **RS** at each range, agree *within one percent* for all ranges out to where the bullet goes subsonic at about 900(+) yards.

Since the inwardly-directed aerodynamic lift force **FL** (earlier simply called **L**) acting on the coning bullet lies in a plane perpendicular to the apparent wind, and since its magnitude **[FL]** is directly proportional to the trigonometric sine of the cone angle **a**, the force **FL** is an attractive central force that is actually also directly proportional to the *radius* **RL** of the bullet's center of mass from a central (neutral) point on the axis of the approaching apparent wind, so that:

$$\mathbf{FL} = -k*\mathbf{RL}$$

A system of this type, a mass subjected to a ***restoring force proportional to its radial displacement***, is termed an ***isotropic harmonic oscillator*** in mechanics. Generally, these radially symmetrical systems produce stable (repeatable or closed) elliptical orbits that are special cases of the strikingly beautiful Lissajous patterns familiar to electronic oscilloscope users. In this particular instance, I expect that bullet spin-stability factors will rapidly ***"circularize"*** any orbital eccentricity in the bullet's coning motion in a fashion similar to the damping of its fast mode oscillation. A circular orbit is just a special case of an elliptical orbit with an eccentricity value of zero. No stable elliptical bullet motions have been reported to my knowledge. One can easily demonstrate an example of an isotropic harmonic oscillator by observing the path of the tip end of a limber fly rod when one moves its handle in a rapid circular motion. Although any central force field can produce circular orbits, I know of only one other system with an attractive central force field that can produce closed elliptical orbits, and that system is the *inverse square law* force field, as is the case with universal gravitation. However, the center of the force field is located at the center of the ellipse in harmonic motion, instead of being located at one focus of the ellipse as with gravitational orbits.

As a *corollary* to this lift-driven coning motion theory, ***the axial direction of the coning motion itself must track the eye of the apparent wind*** throughout the flight of the bullet. [The "eye of the wind" is an old nautical expression meaning "the exact direction from which the wind is blowing."] The coning bullet is responding to a powerful driving force, and it seems to abhor any modulation of its lift force as would occur if its angle of attack (measured from the apparent wind direction) were to vary cyclically during each orbit. The cone axis also defines both the *neutral position* and the *neutral orientation* of the oscillating bullet and must, itself, be parallel to the direction of approach of the apparent wind for stable bullet motion. ***The turning of the bullet's coning axis into the wind is actually the principle resolution of the "excessive lift problem," mentioned earlier.*** This troublesome sideways-acting lift force goes to a net (or average) of *zero* as the ***average direction*** of the bullet's spin axis points directly into the eye of the apparent wind.

Coning Motion Is Inevitable

Even if manufactured and launched perfectly, the bullet *will inevitably commence* a "coning motion" in flight (even though it *may not* have to "*oscillate*" significantly). Coning motion will be immediately initiated by:

1) Any initial or subsequent oscillation of the bullet,
2) Any crosswind at the muzzle or at any point downrange, or
3) Even just the *change in the flight path angle* caused by the inevitable downward curving of the trajectory due to gravity.

As we explained when we were analyzing the BRL data for the epicyclic motion of our example match bullet, the angular amplitude of any initial *oscillation* will be matched by an initially equal *coning angle* **a**. Furthermore, as the oscillation motion damps out, approximately another 50 percent of its initial amplitude will be added into the size of the cone angle **a**. Barring any sudden disturbance, the remainder of the bullet's flight will involve only coning motion. Let me quote Dr. Franklin W. Mann on this subject from page 243 of the 1942 edition of *The Bullet's Flight*:

> *"Most bullets, being more or less unbalanced, begin to develop a tip and an oscillation immediately upon their exit from muzzle, and those that do not tumble in their flight will gyrate, due to air pressure on or near their points."*

The bullet's "developing a tip" means "increasing its angle of attack above zero," and what he refers to as "gyrating" is equivalent to our "coning" motion. Dr. Mann's phrase, "due to air pressure on or near their points," sounds a lot like what we now know to be the aerodynamic lift force that drives the coning motion of the bullet.

Doppler Radar Evidence

We can also "see" the coning motion of the bullets in flight by examining several of the Doppler radar tracks available for small arms bullets. These tracks plot *relative radial velocity* of the projectile versus its *slant range* from the radar unit. The radar units are situated slightly off to one side of the firing range, and the projectiles are launched at reasonably high elevation angles so that they can be tracked for quite some time. The coning motion of the bullet produces the *exaggerated modulation* of the radial velocity component that we see in these plots from the "skin tracking" return, reflected from the afterbody of the coning projectile in flight. This relative velocity modulation is especially significant during the downward arc of the bullet's trajectory where the cone angle is large and the viewing aspect is most favorable for picking up the coning motion.

The Virtual Bullet Concept

The bullet's coning motion with reasonably small cone angles (under 5 or 10 degrees) is ***not incompatible*** with the simpler explanation of wind drift, as discussed in the article, *Understanding Wind Drift*, in the December 2007 issue of *Precision Shooting*. The simple explanation of how wind drift occurs can now be seen to apply quite well to a "virtual bullet" that simply moves its center of mass smoothly along the 3-DOF trajectory, rather than coning around it. In stable flight, the spin axis of the virtual bullet is aligned with the cone axis of the real coning bullet. Moreover, the virtual bullet does not suffer any "coning motion" because, with its nose pointed directly into the apparent wind, it cannot sustain any lift force nor any overturning moment, except perhaps during very limited transient conditions. The virtual bullet has all of the physical characteristics of the real bullet for which it is substituting, except for having a very slightly larger coefficient of drag **CD**. Referring back to Table 1:

$$\mathbf{CD = CD(0) + CD(2)*Sin^2(a)}$$

With this small (third or fourth significant figure) increase in the coefficient of drag, the cross-track component of the virtual bullet's drag force still produces just the horizontal force needed to drift the real bullet by the observed amount. The virtual bullet also spins at the same rate as its real sister bullet at each point along the flight.

Bullet Tracking of the Flight Path Angle

The only coning motion of the bullet that can remain stable in flight is when the motion of the pointing direction of the bullet's spin axis is circular and centered about the eye of the approaching apparent wind as shown in Figure 7. The trajectory arcs downward due to gravity and the bullet encounters variations in the wind environment throughout its flight. The eye of the apparent wind shifts ever lower and often over to one side or the other as the bullet slows and the crosswinds change. The coning bullet points its cone axis directly into the apparent wind, both horizontally and vertically, and "tracks out" both types of variations. But, since the bullet's forward velocity vector (+V direction, tangent to the trajectory) always defines the origin of the familiar "wind axes" used in plotting the bullet's epicyclic motion, the "wind axis" coordinate system rotates downward in pitch along with the bullet's coning axis. Thus, we will never see the bullet's spin-axis motions offset in pitch in a wind axis plot caused by the "pitching over" of the bullet's cone axis to track the changing flight path angle as the trajectory points evermore downward due to gravity. Let me once again quote Dr. Mann on the subject of the bullet's tracking of the curvature of the trajectory (from page 246):

> *"The supposed point-on bullet, therefore, becomes a tipper more and more rapidly, and its axis of gyration …is constantly striving to keep itself in line of the ever increasing trajectory curve…."*

The real bullet tracks either of these types of shifts in the incoming apparent wind direction by selectively enlarging the size of its coning motion whenever the bullet spin axis moves nearer to the eye of the wind. That is, the orbital coning motion "collapses outward" toward its largest angular displacement from the eye of the apparent wind as it becomes circular and stable once again. Unlike the case of a gravitational attraction, the centripetal-acting lift force increases with larger radial displacements from the center of the force field, so that the dynamics of the harmonic orbit are quite different from those of planetary orbits. This line of reasoning also helps to explain why the general case of a wind-centered elliptical orbit is not a stable form of bullet coning motion and why the fast-mode oscillations of the bullet must damp out soon after being initiated. In this way, the average pointing direction of the free flying, coning bullet, and, thus, the direction of the axis of the virtual bullet, rapidly tracks toward the apparent wind direction, even if the real bullet's spin axis never actually points in that direction. And any and all changes in the apparent wind direction during the flight of the bullet will accumulate into the size of the cone angle a. Other bullet designs should damp down the coning angle at a somewhat faster rate between these flight disturbances.

Let me illustrate this "collapsing outward" type of apparent wind tracking by means of a little fable:

> *Say, for example, that one of our example 168-grain bullets, fired in a 600-yard match, is happily "coning" around with a stable cone angle of 2 degrees about the eye of its apparent wind at 400 yards downrange, when it suddenly encounters a radical wind shift—like the complete reversal of a right-to-left 20 knot crosswind—so that the new apparent wind is now approaching from 2 degrees farther leftward than it had been, as seen from behind the bullet. If the spin axis of our coning bullet happens to be pointing near to the new direction of the apparent wind, the bullet's aerodynamic lift force goes nearly to zero! That is, for the moment, the coning bullet's spin axis could be practically aligned with the wind. Or, if the bullet happens to be pointing to the right, its centripetal lift force would be promptly doubled! And our fabulous bullet is still coning around at 34 cycles per second, or one cycle every 54 feet of travel, at 400 yards downrange. Up to this point in the coning motion, the bullet's lift force had been providing just exactly the centripetal force necessary to drive the orbital motion of the bullet's center of mass around in a 2-degree circular coning motion about the mean path of its trajectory. In particular, we can see that when the lift force is suddenly and sharply reduced, the bullet's center of mass, lacking this needed inward force, then "skids out" along a path perhaps similar to a "minimum time" elliptical transfer orbit in a gravitational field, and, within about half of a coning cycle, our bullet establishes a stable new circular orbit with a 4-degree cone angle about the new apparent wind direction (and not something like a 4-degree wide by 1-degree high elliptical orbit). Moreover, the coning bullet's average pointing direction is oriented once again directly into the eye of the (new) apparent wind.*

Smaller changes will happen just as rapidly, but will increase the cone angle by smaller amounts. In fact, the eminent aerodynamicist, Harold R. Vaughn, on page 195 in *Rifle Accuracy Facts*, says that:

> *"It takes less than one fast precession cycle for the bullet to align itself to the relative wind vector and reduce the angle of attack due to the wind to near zero."*

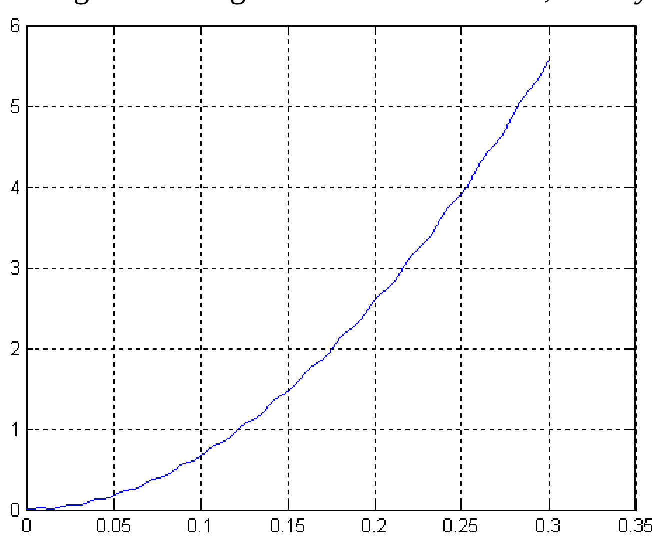
More precisely, it is the *coning motion* of the bullet that aligns its axis with the wind, and such a sudden change in crosswinds would also induce a significant fast-mode (300-hertz) *oscillation* in the bullet's motion. But why complicate such a nice little fable with factual details?

Secondary Drift Effects

Since the virtual bullet has been defined as always moving its axis so as to point into the apparent wind, as with the cone axis direction for the real coning bullet, we can no longer directly use an "overturning moment hypothesis" in explaining the observed secondary drift effects on our match bullets. Both the virtual bullet and the real coning bullet normally fly with ***no angle of attack*** (other than for the stable cone angle **a**) and, therefore, ***generate no net overturning moment***. Also, since the spinning bullet is not really acting as a gyroscope, we cannot directly use gyroscopic responses to explain bullet motions. However, the virtual bullet has the same angular momentum values and moments of inertia, as does the real bullet at corresponding points in the flight. Attempting to change its pointing direction will produce a transient "inertial response" at right angles, as with a similar gyroscopic response. We can envision the virtual bullet as assuming a small temporary angle of attack within the cone envelope of the coning real bullets. Or, more realistically, we might say that after a disturbance in flight the coning bullet can temporarily assume an attitude other than keeping its axis pointed toward the apex of the cone. With further study, the details of how these observed secondary effects are produced should become clearer.

Bullet Spin Drift

Since, as just described, the cone axis of the real coning bullet tracks the eye of the apparent wind, the virtual bullet (which points along the axis of that cone) executes the "arcing over" maneuver as well, so that its axis direction will *track* the tangent to the 3-DOF trajectory as the flight path angle drops below the bore axis direction at launch, and all of the bullets will enter the target nearly point forward even at long ranges (as they are observed to do). Each time the eye of the apparent wind moves *incrementally lower* as the trajectory arcs downward, the real bullet's cone axis direction must *accelerate downward* while the pointing direction of the *clockwise-coning* bullet's nose is ***traversing the right-hand two quadrants of the wind axes*** (the two positive yaw quadrants, in airframe terminology). Of course, this means that, because the bullet is increasing its cone angle incrementally, *the rear of the coning bullet is hung out leftward into the apparent wind-stream by a* **small extra amount** *during this maneuver.* The bullet will experience a very small, discrete *rightward lift pulse* during this transient downward adjustment of the cone axis. This discrete process will recur *once per coning cycle* throughout the flight of the bullet. *Indeed, exactly this type of coning-rate* **modulation** *of the bullet's horizontal "spin drift" can be seen in the fine structure of plots of the bullet's total horizontal drift versus time generated from 6-DOF computer program runs.* Neither of the other two analytical components of the total drift, not ordinary wind drift and not the Coriolis Effect, could be the source of this modulation.



Total Horizontal Drift (inches) vs. Flight Time (seconds) Showing Coning Rate Modulation [Data from 6-DOF run by Bryan Litz]

The “spin-drift” phenomenon was well known back in the late nineteenth century when the militaries of the world thought that long-range volley fire might be effective against massed troops or cavalry from up to 2000 meters away. In fact, the British Army standardized a left-hand twist direction for their service rifles during the nineteenth century so that the resulting leftward “spin drift” would largely cancel the drift due to the Coriolis effect that is always rightward in the northern hemisphere. Dr. Mann rejected the use of the term “drift” for this small, but predictable effect, and referred to it as “trajectory deflection.” He said of it (on page 245 of my 1942 edition):

> *“It is a motion one element of which is skin friction, due to a partial rolling and slipping of the bullet upon increased air pressure on its under side, or that side which is presented toward the center of the earth.”*

I do not personally accept Dr. Mann’s argument on this point. More modern explanations have tended to invoke the overturning pitching moment on the bullet due to the lowering of the direction of approach of the airflow to induce a microscopically small rightward “yaw of repose” (for right-hand twist barrels) as a *gyroscopic response*, which in turn is supposed to produce the observed rightward drift as a *lift force effect*. My little explanation above might make more sense than this classic “yaw of repose” hypothesis, given the powerful coning motion that we now know the spinning bullet must be undergoing throughout its flight. Since the bullet is not responding as would a gyroscope, but instead, *simply points its coning axis in the direction of the apparent wind*, this “yaw of repose” explanation needs to be reexamined.

Vertical Bullet Deflection Due to Horizontal Crosswinds

The perfectly launched, 168 grain *Sierra MatchKing* bullet moving into the uniform, 10 MPH, horizontal, left-to-right crosswind, used as an example in the December 2007 *Precision Shooting* article on wind drift, encounters a 20 MOA leftward shift in the direction of the apparent wind soon after leaving the muzzle. Initially, this example bullet is assumed to be neither coning nor oscillating. The axis of the spinning bullet has to sweep the ***two lower (minus pitch) quadrants*** of the angular pitch and yaw “wind axes” during its leftward acceleration through 40 MOA to begin orbiting clockwise about this new 20 MOA leftward apparent wind direction. During this ***one-time operation***, the ***rear of the bullet is elevated*** at a cone angle increasing in size from zero to the final 20 MOA cone angle (with a similar size, but rapidly damping, fast-mode oscillation added into the cone angle), such that a ***non-recurring downward lift pulse*** slightly deflects the trajectory of the bullet ***permanently downward by a small angle***. BRL terms this type of one-time deflection through a small angle an “aerodynamic jump.” Thereafter, as long as the crosswind remains uniform, our example bullet will continue coning with its cone axis aligned with the apparent wind direction, and with its cone angle slowly increasing as the flight path angle begins to depart from the bore direction at launch. After the downward lifting *transient effect* is completed, the steadily rotating lift force subsequently *cancels itself out*, as far as any trajectory-modifying effects are concerned.

But, let us consider our virtual bullet once more. If we attempt to apply the classic “gyroscopic response” argument to the virtual bullet, we can envision this bullet responding to transient “inertial forces” by flying for a short time with a slight nose-down attitude within the cone being described by the coning bullet, and generating the downward lift pulse necessary to slightly alter the trajectory downward. The only problem with this line of reasoning is that the virtual bullet actually turns ***into*** the wind as if it were responding to a torque that is the ***reverse*** of its overturning moment. For now, I cannot see how to resuscitate this gyroscopic approach to explaining the observed secondary effects.

For Best Accuracy

For best accuracy we should fire the most perfectly balanced examples obtainable of the match-type bullets with the best flight characteristics that we can find, and we need to launch these near perfect bullets with minimum bullet distortion, minimum in-bore yaw, and no yaw tip-off rate due to muzzle crown or bullet base damage or failure of the bullet to obturate correctly. We are striving to *minimize the initial magnitude of the bullet's coning and oscillation angles*. For modern high-velocity, low-drag match bullets, any *in-bore yaw* is immediately multiplied by a factor of approximately 20 to 25 when the bullet first encounters the outside atmosphere, as we know from *Kent's Equation* for calculating the size of the *first maximum yaw* in the bullet's epicyclic motion immediately after launch.

But more importantly, we ***must avoid*** the accuracy-destroying problems of "lateral throw-off" and "aerodynamic jump," to use BRL's terminology. Either of these two latter effects will, quite literally, send a bullet "off on a tangent" right after launch. For example, if one of our bullets were to be statically imbalanced by having its center of mass displaced by just *one ten-thousandth* **(0.0001)** *of an inch* (**RE**) from the axis of the bore during firing, the resulting *lateral throw-off angle* with our rifle's 12-inch twist rate (**TW**) would cause a randomly oriented radial miss distance of **0.188 inches** at 100 yards.

Miss distance = Range*(2*Pi*RE/TW)

[This expression largely explains why target shooters generally try to use the ***slowest twist rate*** that will *(just barely) adequately stabilize their near-perfectly-balanced match bullets*. Harold Vaughn measured several lots of match bullets and found about 1.4 times this amount of offset **RE** to be the typical average static imbalance. But, more encouragingly, he also found that the measured dynamic imbalance of these bullets was truly negligible.]

By using the best available bullets in the best target rifles with barrels having the optimum twist rates and the best chamber and throat designs, ammunition components, and reloading techniques, *we should be able to minimize the occurrence of either bullet oscillations or tangential deflections right out of the muzzle*. These are just some of the *reasons why* we need to keep doing all of the things that most of us are already striving to do for best accuracy.

If all of the above factors are perfectly optimized, the smallest possible diameter at the target distance of the helical path due to bullet coning will be determined by the ***change in the flight path angle*** between bullet launch and impact on the target. This is one area where the flattest shooting rifles have a slight advantage. One can quite easily determine this change in flight path angle by examining "drop tables," as are often printed by 3-DOF exterior ballistics programs. For the conditions used in Table 1, the diameter of the coning motion at 200 yards, for example, could be reduced from a typical **0.088-inch** to as little as **0.010-inch** if everything but the change in flight path angle could be eliminated (i.e., if there were no wind at all, and we fired only perfectly made bullets of this same type, and launched each of them perfectly). Of course, for most of our competition bullets, we simply do not have the detailed flight data with which to make these computations.

Summary

We discussed the elements of the standard formulation of bullet motions in flight and the fact that it does not include provisions for handling ambient winds, which leads to a serious flaw in that rationale, the "excessive lift problem." Then we presented the basis of a new formulation of the bullet's flight motions, built around the bullet's "coning motion" in flight. This motion is driven by the lift force due to its cone angle, and, thereby, uses up that lift force and resolves the aforementioned problem. We showed that the coning motion is an isotropic harmonic oscillation about the axis of the approaching apparent wind, and mentioned that its rate of oscillation is independent of the amplitude of its motion. We argued that the axis of this coning motion strives always to point toward the eye of the apparent wind, thereby incorporating wind handling into the new formulation. We presented the concept of a "virtual bullet" that flies smoothly along the path of the trajectory without coning and points its nose along the cone axis and into the wind. We pointed out that the virtual bullet produces the observed wind drift due to crosswinds and that it also "arcs over" to follow the downward curvature of the trajectory. Then we outlined a "coning bullet explanation" for the two secondary bullet drift phenomena that we can observe. We also pointed out that much work remains to be done to complete this new formulation of bullet motion.

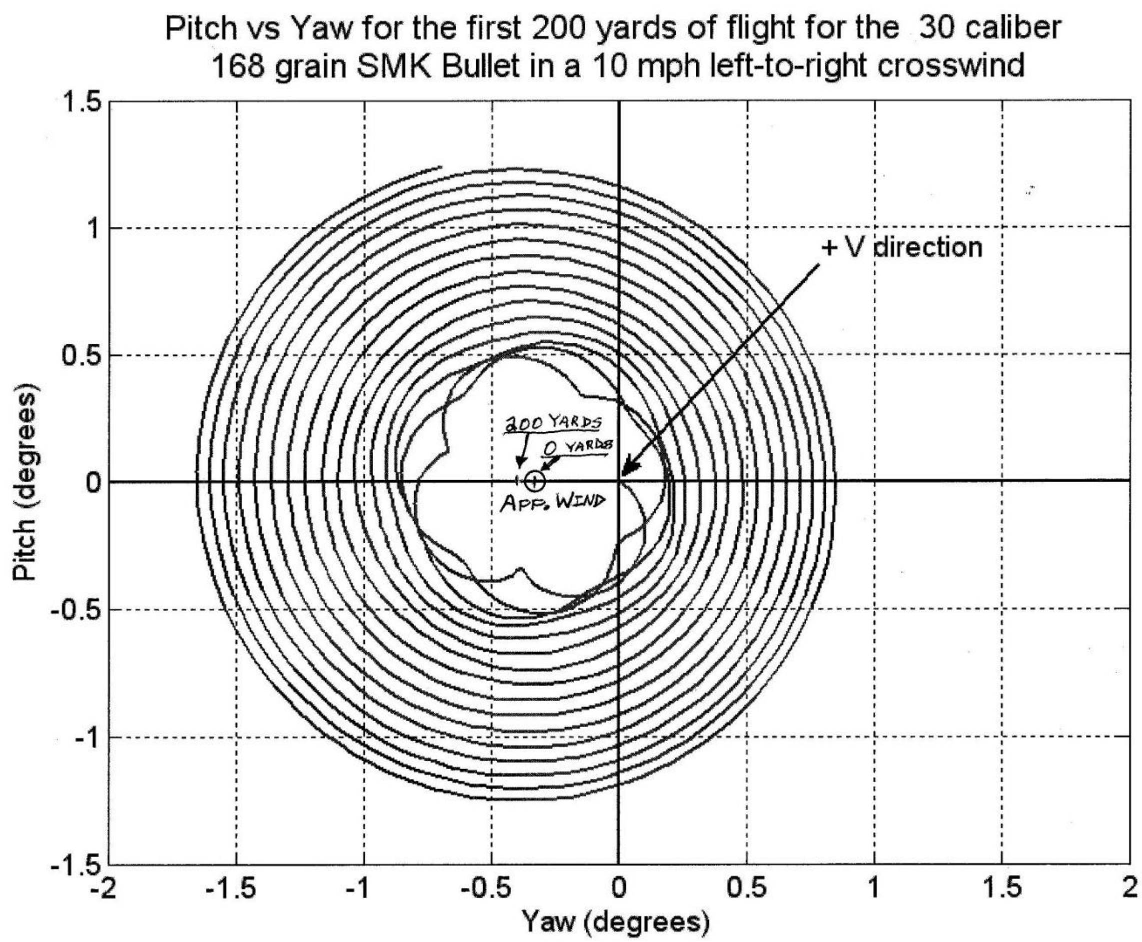
Addendum to Part II

I recently requested Bryan Litz to run exactly our example case here through his 6-degree-of-freedom (6-DOF) computer program, modeling bullet motions that he has developed based upon Chapter 9 of Robert L. McCoy's book, *Modern Exterior Ballistics*. Bryan graciously consented and produced a successful 6-DOF model run for the first 200 yards of the simulated flight of a perfectly made and perfectly launched 30 caliber, 168 grain Sierra International bullet starting out at 2600 FPS through a uniform 10 MPH left-to-right crosswind in an ICAO standard sea-level atmosphere.

For each 0.0001-second timestep of the whole time-of-flight, this 6-DOF program first calculates the attitude (roll, pitch and yaw) of the simulated bullet based on the *aerodynamic moments* that apply during that time increment, and then, using these *three* calculated attitude angles, the program computes the *three* earth-fixed position coordinates of the bullet from the *aerodynamic forces* that apply during that same timestep. [Hence, the term ***six degree-of-freedom***.] In order to run properly, the program requires input of the complete specification of the mass characteristics of the bullet itself, the initial conditions of the simulated bullet's flight (including its initial linear and angular rates) and of its atmospheric environment, as well as input of the aerodynamic force- and moment-defining coefficients, unique to the subject bullet type, over at least the full range of Mach numbers that may be needed in the run. With the correct input data, this type of program can compute bullet motions that *agree outstandingly well with our observations of real bullets in flight*.

> ***Important Note:*** This *non-analytical* 6-DOF program does not know anything about, nor even compute, the *analytical formulations that we have been discussing in this article* for such things as oscillation and coning magnitudes and rates, or wind drift, or yaw-of-repose, or the radius or axial direction of the bullet's coning motion. Nor does the program compute the smooth point-mass, 3-DOF trajectory that we have always found so useful as an analytical tool. To the extent that our analytical formulations *correctly and completely explain* bullet motions, they are *all combined (or summed) together* in the outputs of the 6-DOF program runs. [In fact, separating out the different analytical effects from measurement data streams of this type, wherein they are all "boiled together," is exactly the task of the engineering analyst, including yours truly.]

The illustrated clockwise-spiraling plot was produced by Bryan's 6-DOF program and shows for each timestep the pointing direction of the bullet's spin axis plotted against the usual *wind axes*, of pitch versus yaw (in degrees). At launch, the "**+V** direction" of the velocity of the center of mass of the simulated bullet is pointing directly toward the *origin of the wind axes* (as it ever will be), the spin-axis of the bullet is initially pointed at this origin, and the bullet is *neither coning nor oscillating* in this idealized firing. Notice how the simulated bullet ***initiates both coning and oscillation immediately upon encountering the 10 MPH crosswind just out of the muzzle***. [Muzzle blast effects are not simulated.] The first *coning cycle* rapidly centers itself about the *eye of the apparent wind* that I have annotated at 0.323 degrees (19.4 MOA) left of the origin. As the bullet slows in its flight, the eye of the approaching wind migrates even farther leftward to 0.380 degrees (22.8 MOA) at 200 yards downrange (approximately where this run was terminated). Notice that, as mentioned earlier, the change in pitch attitude of the coning motion due to "arcing over" to follow the dropping trajectory does *not* show up in this "wind axes" plot. ***The coning axis always points directly into the eye of the apparent wind.*** Also notice that the first motion of the bullet's spin-axis is immediately to point its nose strongly downward. This rapid initial motion of the bullet's nose helps to explain the one-time, permanent, *downward angular deflection* of the bullet's trajectory that we know occurs upon the bullet's first encountering a purely horizontal left-to-right crosswind.



This mathematically simulated bullet hits the 10 MPH crosswind so abruptly (just out of the muzzle) that it starts to *oscillate*, in addition to coning, in response to this disturbance to its flight. One can count approximately 3.7 of the rapidly damping, fast-mode oscillations occurring during the first slow-mode coning cycle. When one adds to this value the one oscillation cycle that we get "free" with each coning cycle (due to the way these motions were defined), we get the expected value of about 4.7 for the ratio of the rates of these two spin-axis epicyclic motions during the first coning cycle (the first 14 yards of flight distance). [Table 1 shows that this ratio should be 4.58 fast-mode cycles per slow-mode cycle at the muzzle, increasing to 5.54 at 100 yards.] After the high-rate oscillations damp out (to non-detectability after about four to five coning cycles), the coning motion is essentially ***circular***, even though the cone angle slowly spirals out to 1.25 degrees at 200 yards, even in this perfectly launched, ideal example. This increasing cone angle is what we meant by saying that this particular bullet is somewhat *dynamically unstable*. ***This single figure, output from one independently conducted simulation run of a non-analytical computer model, goes far toward demonstrating my claims in this article.***