The Bullet’s Flight through Crosswinds

Part I

By James A. Boatright

Introduction

This article is an attempt to explain some important aspects of the modern rifle bullet’s flight through the atmosphere in straightforward English without using very many mathematical equations. I hope to provide a broader and deeper understanding of the complex in-flight motions of our best match-type bullets fired through real crosswinds from our most accurate target rifles. This work follows on where my last article, “Understanding Wind Drift,” in the December 2007 issue of *Precision Shooting Magazine* left off. In that article we discussed horizontal wind drift due to crosswinds. This effect was discussed as if our bullets always flew straight, point-forward through the air. We could explain how the rapidly clockwise spinning bullet, acting similar to a toy gyroscope, always moves its spin axis at right angles to any overturning moment (torque). It would not be much of a stretch for us to realize that those motions could continue on around and become a clockwise gyroscopic precession-like “coning” motion. In fact, until recently I still thought that the bullet’s coning motion actually *was* gyroscopic precession, with a gyroscopic nutating motion superimposed, as has been the common wisdom for many years. After studying the problem for several months, I have concluded that the coning motion of a bullet is no more than a cousin to ordinary gyroscopic precession. We shall examine the causes and effects of this *coning motion* in some detail, including presenting new “coning motion” explanations for each of the three observed ballistic phenomena that are usually explained as “gyroscopic effects.” These phenomena include: (1) The “arcing over” of the bullet to follow the flight path angle of the trajectory that is usually explained as a gyroscopic precession effect, (2) The well-documented vertical displacement of the bullet due to flying through a purely horizontal crosswind that is normally said to be a Magnus effect, and (3) The long range spin-drift of the bullet that is usually said to be caused by aerodynamic lift due to a small sideward “yaw of repose.” We will also indicate why we went to so much trouble in the *Precision Shooting* articles on “*The Well Guided Bullet*” in September and October 2006 to explain how to launch our precision-made, long range match bullets with the least possible amount of in-bore yaw.

We will discuss some details of the measured flight characteristics of the old 30 caliber *Sierra* 168 grain International bullet, the direct ancestor of today’s 168 grain *Sierra MatchKing* bullet. The only reason that we know anything about this obsolete bullet is that the Ballistics Research Laboratory (BRL) at the US Army Aberdeen Proving Ground in Maryland measured and documented the flight characteristics of this and other match bullets back in the 1980’s. We will switch from using the old US Army standard METRO atmosphere to the International Civil Aviation Organization (ICAO) standard atmosphere used by BRL in these bullet studies. The data, formulae and information used in this study predominantly come from *BRL-MR-3733, The Aerodynamic*

Characteristics of 7.62MM Match Bullets, December 1988, by Robert L. McCoy and from his important book, *Modern Exterior Ballistics,* 1999, Schiffer Military History, Atglen PA. The BRL Memorandum Report is freely available online at http://stinet.dtic.mil/. Robert L. McCoy's ballistics book (often simply referred to as the *MEB*) is currently available for $95 at www.schifferbooks.com. I believe my ideas about bullet behavior in flight to be congruent with the observations and, for the most part, with the conclusions of Dr. Franklin W. Mann as documented in his *magnum opus* of 1909, *The Bullet's Flight From Powder To Target*, which only became comprehensible to me as I thought more deeply about the bullet's coning motion and as I learned to interpret his fluently used nineteenth century syntax. I will include several relevant quotes from *The Bullet's Flight* so that you can see what I mean. Even working more than a century ago, Dr. Mann has still studied bullet motions in flight more extensively and more intensively than has any other person, before or since.

This article is largely an attempt to explain what has been known for years in new and more understandable ways and is not reporting any new research. I will start in *Part I* of this study by describing the standard analytical formulations for the major forces and moments affecting the bullet and for the epicyclic motions of its pointing directions in flight. I will present the analytic formulations used at BRL, as documented by Robert L. McCoy, in Chapters 10, 11 and 12 of his book. These formulations are simplified, "linearized" versions of the full equations of motion that would be used in a 6-degree-of-freedom (6-DOF) computation of a projectile's motions in free flight. One of the simplifying assumptions used in these chapters is that there is ***no wind of any kind***, so that air drag is due to bullet velocity alone. I will end *Part I* with an introduction to the serious "excessive lift problem," that I believe is not properly handled within the standard formulation.

Then, in *Part II*, I will discuss at length the bullet's "coning motion" at the heart of an expanded formulation for bullet motion in flight. This discussion will include perhaps some new insight into the dynamics of this coning motion as a ***lift-force driven, isotropic harmonic oscillator***, and why the axis of this cone is constantly driven to point directly into the eye of the apparent wind throughout the bullet's flight. I will include a discussion of the important "excessive lift problem," and how it can be addressed within the new formulation outlined here. I will elaborate upon the subject of how we know that coning motion really must occur with each rifle bullet fired. And, finally, I will define an equivalent "virtual bullet" that flies along the mean, point-mass (3-DOF) trajectory, but without coning, and is always oriented point-first into the air-stream. This virtual bullet concept validates the "wind axis" coordinate system centered on the moving center of mass as used by BRL and most other ballistics researchers. As fringe benefits of all of this rigmarole, we gain what may be perhaps the first logically consistent explanation of how the bullet's pointing direction "tracks" the downward-curving trajectory, how the bullet drifts rightward at long ranges, and how the bullet moves vertically on the target in response to purely horizontal crosswinds.

Part I. The Standard Formulation

Forces and Moments

The major forces and moments acting on our example 30 caliber, 168 grain, Sierra International bullet have been calculated in an Excel spreadsheet at 100-yard range intervals from the muzzle to 1000 yards. The bullet is launched at 2600 FPS into a sea level ICAO standard atmosphere with a pressure of 760mm, or 29.92 inches, of mercury, a temperature of 59 degrees Fahrenheit, and zero humidity, for an air density of 0.0764742 pounds per cubic foot. This bullet performs the typical fast-mode and slow-mode epicyclic pointing motions, as measured in the Spark Photography Range at the BRL. And Robert L. McCoy was able, by finding suitable *initial conditions*, to duplicate these epicyclic motions numerically using a 6-DOF computer model for this bullet. These major forces and moments are calculated here based on BRL data, using their standard analytical formulations, and are given for certain key ranges in Table 1, Studies of 308 Sierra International Bullet in ICAO Atmosphere. The spreadsheet is freely available for downloading and use by those interested in experimenting with these calculations.

Air Drag Force

In the standard formulation, the size of the *drag force*, acting to retard the forward motion of our example bullet at any point in its flight, is calculated as the product of the *dynamic pressure* exerted by the *atmosphere* on the *bullet* at their relative velocity, times the bullet's *cross-sectional area*, times a suitable *coefficient of drag* (**CD**) value. The dynamic pressure is one half the mass density of the atmosphere times the square of the bullet's velocity through the air. When multiplied by the cross-sectional area of the bullet, this dynamic pressure yields a kind of maximum potential dynamic force that might retard a "non-streamlined" bullet. This potential dynamic drag force (**DYN**, in pounds) is incorporated into the drag computation by BRL for several good reasons:

1) The dimensions and units work out correctly to yield a meaningful force.
2) The explicit dependence upon atmospheric density allows us to correct for subsequent variations in that parameter.
3) This "straw-man" drag force can be multiplied by a particular value of a dimensionless coefficient of drag **CD** value to produce the actual measured drag force in a given set of conditions.
4) The **CD** values for a given bullet can be specified as a complicated function of several flight variables, including the bullet's Mach number (airspeed) and its total angle of attack **a**.

This well proven approach to modeling the mathematics of a complicated physical function is used by BRL in accordance with standard practices often used in engineering. We should point out that the actual drag force experienced by the bullet interacting head-on with the ambient atmosphere at supersonic speed comprises several different types of drag forces that can be analyzed separately: 1) nose, or ogive, drag, 2) skin friction, and 3) base drag. Spark shadowgraph photography of actual bullets in supersonic flight show beautiful images of several conical shock waves and boundary layers of laminar and turbulent flows.

Table 1. Studies of .308 Sierra 168gr International Bullet in ICAO Atmosphere

Range	yards	Muzzle	100	200	400	600	900	1000	Yards
Time of Flight, t	seconds	0.0000	0.1200	0.2501	0.5461	0.9033	1.5899	1.8613	seconds
Bullet Velocity, V	ft/sec	2600.0	2403.2	2214.9	1852.0	1524.4	1148.2	1069.6	ft/sec
Mach Number	(A=1116.45 ft/sec)	2.329	2.153	1.984	1.659	1.365	1.028	0.958	
Spin Rate, p	cycles/second	2600	2558	2516	2428	2337	2192	2142	hertz
Spin/cal (pd/V)	rad/caliber	0.1613	0.1745	0.1863	0.2153	0.2521	0.3147	0.3305	rad/cal
Cone Angle (a)	degrees	1.018	1.54	1.7	2	2.3	5	5	degrees
Del=Sin(Cone Angle, a)		0.0178	0.0269	0.0297	0.0349	0.0401	0.0872	0.0872	
Aeroballistic Coefficients (from BRL Data):									
Coef of Drag, CD(0)		0.331	0.34	0.353	0.374	0.412	0.446	0.41	
Drag Corr, CD(2)		4.96	5.58	6.1	7.2	7.18	3.08	2.9	
CD=CD(0)+Del^2*CD(2)		0.3326	0.3440	0.3584	0.3828	0.4236	0.4694	0.4320	
Coef of Mom, CM(a0)		2.63	2.71	2.78	2.95	3.07	3.2	3.42	
Mom Corr, CM(a2)		-4.4	-4.4	-4.4	-4.4	-4.4	-4.35	-4.3	
CM(a)=CM(a0)+Del^2*CM(a2)		2.6286	2.7068	2.7761	2.9446	3.0629	3.1670	3.3873	
Coef of Lift, CL(a)		2.75	2.65	2.58	2.36	2.11	1.46	1.31	
Spin Damp, CL(p)		-0.00708	-0.00735	-0.00750	-0.00820	-0.00889	-0.00990	-0.01030	
Calculated Forces and Moments:									
Dynamic Force, DYN	pounds	4.16	3.55	3.02	2.11	1.43	0.81	0.70	pounds
Drag Force=DYN*CD	pounds	1.3830	1.2223	1.0815	0.8076	0.6055	0.3807	0.3041	pounds
Lift Force=DYN*CL*Del	pounds	0.2032	0.2530	0.2310	0.1738	0.1211	0.1032	0.0804	pounds
F = SQRT(D*D+L*L)	pounds	1.3978	1.2482	1.1059	0.8261	0.6175	0.3944	0.3145	Pounds
L/D Ratio		0.147	0.207	0.214	0.215	0.200	0.271	0.264	
Angle b=ARCTAN(L/D)	degrees	8.36	11.70	12.06	12.14	11.31	15.17	14.80	degrees
R = CP-CG = CM(a)/(CD+CL(a))	calibers	0.853	0.904	0.945	1.074	1.209	1.641	1.944	calibers
Moment=(d/12)*DYN*Del*CM(a)	lbs-ft	0.004985	0.006634	0.006379	0.005565	0.00451	0.005746	0.005333	lbs-ft
Computed Tri-Cyclic Motions:									
P = (Ix/Iy)(pd/V)	radians/caliber	0.021645	0.023417	0.024998	0.028891	0.033835	0.042242	0.044363	rad/cal
M = (ky^-2)CM*a	(rad/cal)^2	6.89E-05	7.1E-05	7.28E-05	7.73E-05	8.04E-05	8.38E-05	8.96E-05	(rad/cal)^2
Fast Mode Oscillation Rate	(cycles/sec)	286.4	295.6	297.1	297.5	295.5	285.9	280.2	cycles/sec
Slow Mode Coning Rate (CR)	(cycles/sec)	62.51	53.32	46.23	34.25	24.31	14.86	14.07	cycles/sec
Ratio of Rates	(Fast/Slow)	4.58	5.54	6.43	8.69	12.15	19.24	19.92	
RL=ky^2*(CL/CM)*Ratio*Del	inches	0.021	0.036	0.044	0.060	0.084	0.192	0.167	inches
Apex Dist=RL/(d*Del)	calibers	3.87	4.38	4.82	5.62	6.76	7.16	6.22	calibers
Calculated Cross-Check Data:									
Torque=(d/12)*R*F*Sin(a+b)	lbs-ft	0.004984	0.006632	0.006377	0.005563	0.004507	0.005729	0.005318	lbs-ft
[Matches "Moment" data above.]									
RS=(3*g/(Pi^2*w))*L/CR^2	inches	0.021	0.036	0.044	0.060	0.083	0.190	0.165	inches
100*(RL/RS-1)=Percent Delta		0.05	0.12	0.14	0.18	0.23	1.04	0.96	percent
[Matches "RL" Orbit Radius data above.]									
Gyro Nutation Rate=(Ix/Iy)p/2Pi	hertz	348.96	343.33	337.69	325.86	313.65	294.24	287.54	hertz
[Do NOT Match Fast Mode Oscillation Rates above.]									
Gyro Prec Rate=Mom/(2Pi*p*Ix)	hertz	0.91	1.23	1.21	1.09	0.92	1.25	1.18	hertz
[Do NOT Match Slow Mode Coning Rates above.]									

The coefficient of drag (**CD**) for a given bullet is a function that varies significantly with the bullet's Mach number (bullet speed divided by the speed of sound, or 1116.45 FPS for our sea level ICAO atmosphere) and increases very slightly with the angle of attack **a** of the bullet in flight. As shown in Table 1, the actual drag force (in pounds) on our example bullet *decreases continually* throughout the supersonic portion of its flight even though the **CD** values are *increasing* as the bullet slows. The drag force on the bullet in supersonic flight only varies approximately with the *three halves power* of the velocity of the bullet through the air. It is important to note that the ***direction*** of the aerodynamic drag force does ***not*** depend in any way upon the free-flying bullet's *attitude,* or orientation in space. By definition, air drag always acts in a ***downwind*** direction, which means ***in the direction of apparent motion of the undisturbed local air mass as seen from the perspective of the moving bullet itself at any instant during the flight.*** The drag force on the bullet and the weight of the bullet are the only two forces used in the commercially available trajectory programs, such as *Sierra's Infinity* software package, to compute the bullet's three-degree-of-freedom (3-DOF) trajectory.

Apparent Wind Concept

A good way of defining the direction in which the drag force acts is to re-introduce the concept of the "apparent wind direction," as mentioned in the previous article. Referring to the diagram in Figure 1, we can determine the ***apparent wind velocity vector*** at any instant during the bullet's flight by forming the vector difference: ***true wind velocity minus bullet velocity***.

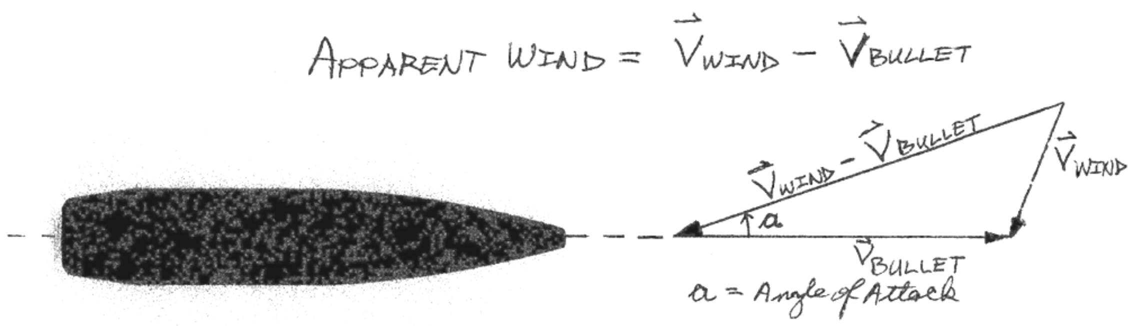

FIGURE 1. APPARENT WIND CONCEPT

These two instantaneous wind and bullet velocity vectors are each three-dimensional vectors specified in the same earth-fixed coordinate system. The resulting three-dimensional apparent wind velocity vector is with respect to the moving bullet. Note that the instantaneous direction of the *bullet's velocity vector* completely defines the principle direction (origin) of the "wind axes" in the standardly-used coordinate system centered on the bullet's mean center of mass (or gravity, MCG), which moves along the 3-DOF trajectory with the bullet. ***The drag force vector acting on the bullet is in the direction of the apparent wind velocity vector.*** If there truly is ***no wind***, e.g., when firing in an indoor range such as the BRL Spark Photography Range, ***the apparent wind is the same as the negative of the bullet velocity vector***. And, since all BRL data on bullets in free flight is collected in this way, this "no wind case" is exactly why all known diagrams of bullet pointing motions have shown epicyclic patterns ***centered about the origin of the "wind axes" aligned with the bullet's velocity vector direction, instead of about the incoming direction of the apparent wind***. The standard analytical formulation fits ***only*** indoor spark range data, and it cannot accommodate real winds. [We intend helping to remedy that situation.] Also, note that the apparent wind velocity vector is normally going to be controlled predominantly by the much larger bullet velocity compared to the wind velocity in any reasonable case except, perhaps, for air-to-air or air-to-ground gunnery. The concept of apparent wind is an important key to understanding the flight of the bullet.

Overturning Moment

In the standard formulation, as used at BRL, the overturning moment acting on the bullet at its CG is found in a way similar to how we just found the drag force. The moment (**M**, in pounds-feet) is the product of the potential dynamic force (**DYN**, in pounds) times the diameter of the bullet (**d**, in *feet*), times the Sine of the angle of attack **a**, times a dimensionless moment coefficient **CM(a)**. The overturning moment coefficient for each bullet varies primarily with Mach number, and secondarily with angle of attack **a**.

Neglecting the bullet's "spin effects" for now, a ***single force vector*** applied at the "center of pressure" on the axis of symmetry of the bullet can represent the entire ***distributed aerodynamic force*** exerted over the whole surface of the bullet at any point in space and time during the bullet's flight. In fact, the center of pressure (CP) is an artificially designated point on the bullet's axis of symmetry, normally located well forward of its center of mass, at which this *total aerodynamic force vector* would have to be applied to produce *both* the instantaneous *net force* and the instantaneous *overturning moment* acting on the bullet. To avoid possible confusion, we will use CG, for center of gravity, as an abbreviation for the center of mass of the bullet. A basic principle of the physics of rigid bodies is that an applied force such as the total aerodynamic force **F** through the bullet's center of pressure (located at a vector distance **R** along the axis of the bullet, forward from the bullet's center of mass) is equivalent to, and can be replaced by, the same force **F** acting through the bullet's center of mass plus a *force couple* (**F** and **–F**) in the common plane producing only a *moment* about the center of mass. The moment about the bullet's CG is defined to be **RxF, a vector cross product**. A moment is a torque vector, the rotational analogue of a force vector in an inertial space. The direction of the torque vector is the axis of the rotation about the CG and is perpendicular to the plane of **R** and **F**, and positive in the usual right-handed sense. The scalar size [indicated by enclosure in brackets] of this vector overturning moment is the magnitude (in pounds-feet) of the vector cross product, and is given by:

[M] = [Overturning Moment] = [RxF] = [R]*[F]*Sin(a+b)

where **R** = Vector distance displacement of the CP from the CG in feet

[R (in calibers)] = CM(a)/(CL(a) + CD), using BRL coefficients.

F = **D + L** = Total aerodynamic force vector in pounds, and

a+b is the angle between the directions of the two vectors, **R** and **F**, as shown in Figure 4.

The current total angle of attack is **a**, and **b** is the angle whose tangent is the current lift-to-drag ratio **L/D**. In the standard formulation, the tangent of the angle **b** can be found from **CL(a)*Sin(a)/CD**, where **CL(a)** is the dimensionless coefficient of lift, discussed below.

Figures 2, 3 and 4, respectively, show the drag force **D** and lift force **L**, acting separately on the bullet's center of pressure CP; then the total (vector summed) aerodynamic force **F**, acting at the CP; and finally the total force **F**, acting at the bullet's center of mass CG, plus the force couple producing only a moment about the CG, and no net translational force. All three of these diagrams are ***mechanically equivalent***. [It may help to see that

Figure 4 is equivalent to Figure 3, if we envision adding into Figure 3, the two *offsetting* forces (**F** and **–F**) acting at the CG of the bullet.] We prefer to analyze the bullet's motions in terms of the forces and moments acting on the center of mass CG of the bullet where they are "de-coupled" from each other.

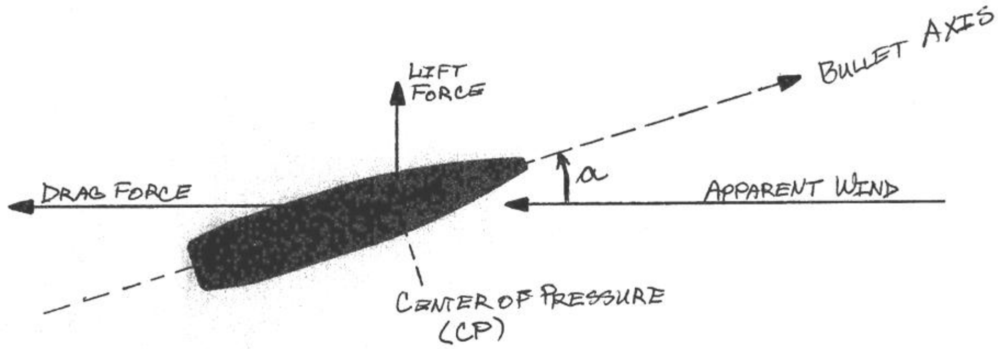

FIGURE 2. AERODYNAMIC FORCES ON BULLET IN FLIGHT

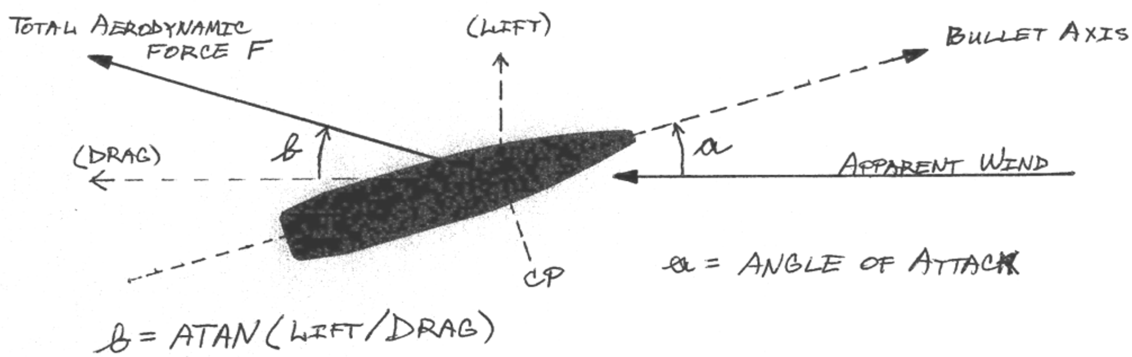

FIGURE 3. TOTAL AERODYNAMIC FORCE

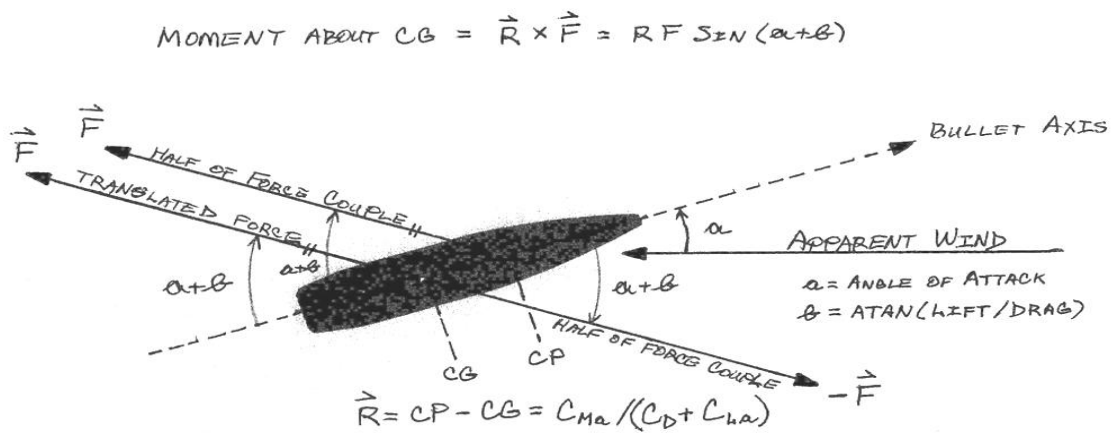

FIGURE 4. TRANSLATED FORCE AND MOMENT ABOUT CG

Of course, the displaced total aerodynamic force **F** shown in Figure 4 can readily be resolved once again into its lift **L** and drag **D** components, only this time acting upon the bullet's center of mass.

The effects of this translated lift force **L** acting on the CG of the bullet will be extensively discussed in the remainder of this article. The values for the BRL-formulated overturning moment are shown for selected ranges in Table 1, along with my crosscheck values calculated as above, but labeled as "torque" values, for distinction. The two sets of moments agree to the fourth significant figure out to long range.

Lift Force

In the standard analytical formulation, the lift force **L** acting perpendicular to the drag force, and at the CG of the bullet in flight, is found in a way analogous to how the other forces and moments on the bullet are evaluated. The lift force (in pounds) is the product of the potential dynamic force (**DYN**, in pounds) times the Sine of the angle of attack **a**, times a dimensionless coefficient of lift **CL(a)**. The coefficient of lift for each bullet varies with the Mach number of the bullet in flight (and *not* with the angle of attack in the basic analysis).

If the single, total aerodynamic force vector **F** shown in Figure 3 is perfectly aligned with the bullet's *apparent wind direction,* a "purely ballistic drag" will be produced. Otherwise, the difference between the downwind *ballistic drag force* component and the *total aerodynamic force* is a rectangular force component ***taken to be perpendicular to the drag force (and, thus, perpendicular to the apparent wind direction, as well)***. This aerodynamic force component is termed ***lift*** regardless of the radial direction (roll angle) at which the side-force is acting to pull the bullet away from, or toward, its 3-DOF ballistic trajectory. For rotationally symmetric projectiles, such as our example bullet, the "roll orientation" of the lift force vector in the plane perpendicular to the drag vector *is completely determined* by the roll orientation of the bullet's ***yaw angle***, or "total angle of attack." In my nomenclature, the term "ballistic," as in "purely ballistic drag," implies *zero lift* when used it in this context. More generally, a "ballistic trajectory" is that of an *un-guided and usually un-powered* projectile, as in the *absence* of any intelligent control over the size and direction of the *lift vector* of the projectile, or over any available *thrust* *vector*, either or both of which might otherwise be used to steer the projectile toward a particular target, for example. The magnitude of the side-force vector called "lift" is inherently positive, without regard to its orientation with respect to the direction of the local gravity gradient.

The ***magnitudes*** of both the lift and drag forces ***certainly do depend upon the total angle of attack (or yaw)*** of the bullet in flight. If the bullet flies exactly point-forward into the local air mass, it produces its smallest possible drag force and no lift at all. Any other bullet attitude in flight is characterized by a single yaw angle (**a** as shown in Figure 2) measured from the direction of the *approaching apparent wind* to the direction of the *bullet's axis of symmetry* (nominally its spin axis). This identification of the "total angle of attack" of the flying bullet with a "yaw angle," is used in studying the exterior ballistics of rotationally symmetric, spin-stabilized projectiles (e.g., bullets in free flight), and differs from the airframe-oriented definitions of the Eulerian attitude angles; "pitch," "roll" and "yaw" used in aircraft flight control. At least for reasonably small yaw angles **a**, the size of the bullet's ***aerodynamic lift force is modeled as being*** **linearly proportional** ***to the sine of the yaw angle a*** producing it. And, both the bullet's coefficient of drag **CD** and its overturning moment coefficient **CM(a)** are modeled as having small *additive correction terms* that each vary with the ***square of Sin(a)***. In this way, the bullet's drag force and moment coefficients are corrected to increase and decrease, respectively, by small amounts with increasing yaw angle **a**. The lift force, like the drag force and the overturning moment, is not directly dependent upon the spin of the bullet. The lift forces on our example bullet are shown in Table 1 for selected ranges, as they would occur during a typical flight.

Other Forces and Moments

The Spin (or Roll) Damping Moment, Pitch Damping Moment, Magnus Force, Magnus Moment, etc., are not explained here because an understanding of them is not really essential to this level of discussion.

Epicyclic Swerve

We can study the bullet's motions in flight by firing through "yaw cards" at a shooting range, by using spark photography of free-flying bullets inside a fully instrumented test range, by tracking the projectiles on an outdoor firing range using a modern Doppler radar system, and by utilizing sophisticated six-degree-of-freedom (6-DOF) exterior ballistics computer modeling programs using coefficients derived from the instrumented firing ranges or from measurements made with a physical model of the bullet in a supersonic wind tunnel. Each of these approaches produces evidence showing that the spin axes of real rifle bullets actually trace out an *epicyclic motion*, rotating clockwise as seen from behind, when the bullets are fired from right-hand twist barrels. Here we are discussing modern, high-velocity, low-drag, jacketed target rifle bullets in particular, but one could apply these concepts reasonably well to any spin-stabilized, rotationally symmetric projectiles of more than about two calibers in length.

The term "epicyclic" refers to the path of a point somewhere along a radial of one circle rolling in a plane along the outside of a second fixed circle without slipping. In the terminology of BRL, the second, relatively fixed circle is called the "slow-mode" motion, and its radius is called the "slow-mode arm." As shown in Figure 5, the center of the "fast-mode" circle is located at the rotating end of the slow-mode arm. Its "fast-mode arm," usually rotating several times faster than the rate of the slow-mode arm, traces the "epicyclic path" of the pointing direction of the bullet's spin axis (as shown in Figure 6). In this case the direction of the spin axis initially follows an epicyclic path in pitch and yaw angles (reverting to airframe-based terminology for the moment), that is centered on the bore pointing direction of the rifle barrel's muzzle at the very beginning of the bullet's flight. As the bullet moves down range, the direction indicated by the *origin of the pitch and yaw axes* always points in the direction of the *bullet's velocity vector* (i.e., tangent to its mean 3-DOF trajectory). We term these angular pitch and yaw coordinate axes as the ***wind axes*** for short. This coordinate system definition is convenient for conceptualizing the complex responses of the spinning bullet to the aerodynamic forces and moments affecting its motion as a free-flying body.

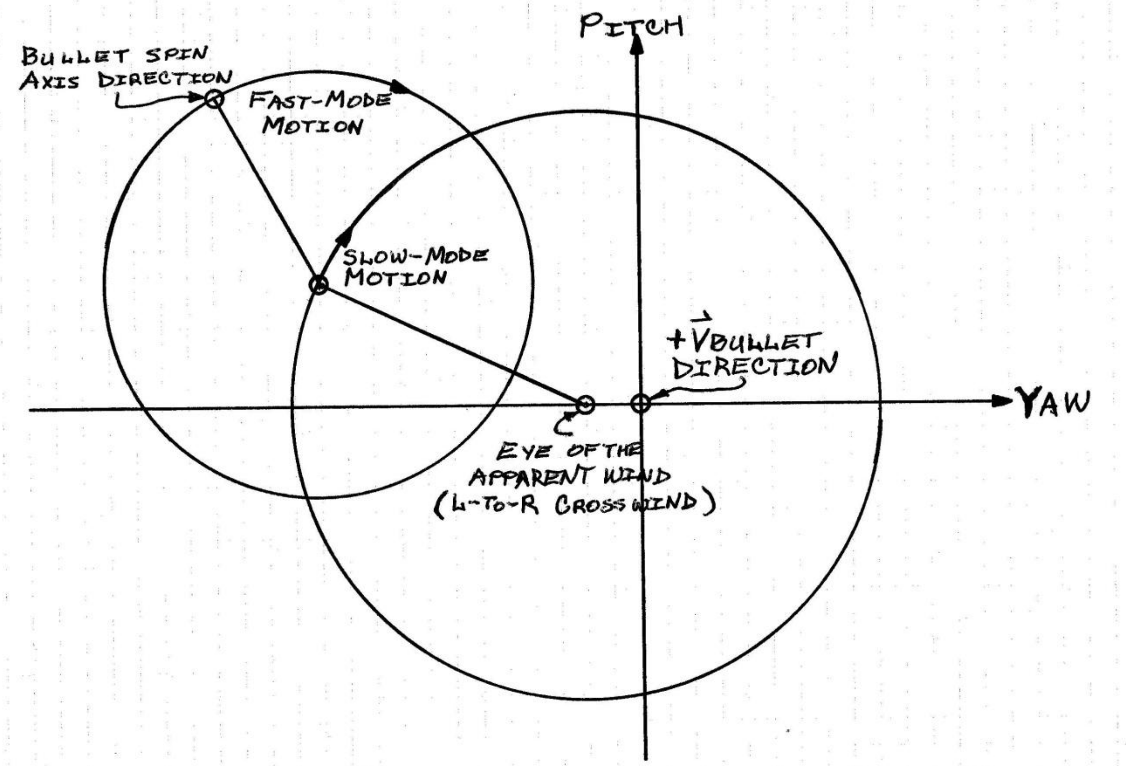

FIGURE 5. SLOW-MODE AND FAST-MODE MOTIONS.

In the epicyclic pattern of movement of the bullet's pointing direction immediately after launch, the two clockwise motions (shown as slow-mode and fast-mode motions in Figure 5) are essentially independent of each other, once set into motion, and are damped at differing rates. While the bullet's spin axis does trace an epicyclic pattern as shown in Figure 6, unfortunately the BRL terminology, "epicyclic swerve," could misleadingly imply that some physical object, such as the bullet's CG, is following an epicyclic path, which it cannot do. Only the *pointing direction* of the bullet's spin axis can follow such an intricate path in angular pitch and roll coordinates. I prefer simply to describe the motion of the bullet itself as "coning," which accurately names both the motion of the bullet's CG and the stable, slow-mode movement of the pointing direction of the bullet's spin axis, as shown in Figure 7, Bullet Coning Motion.

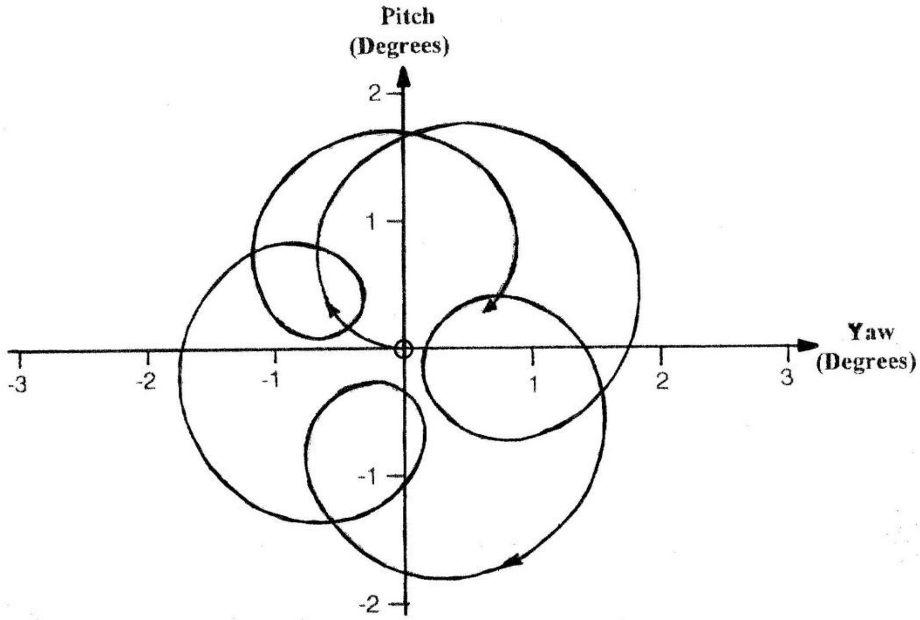

FIGURE 6. EPICYCLIC MOTION (MUZZLE TO 15 YARDS).
[AFTER R.L. McCOY, MODERN EXTERIOR BALLISTICS, SCHIFFER, 1999]

BRL Data on Bullet Epicyclic Motions

Without going into all the gory details, let me just state that the auxiliary parameters and the fast and slow rates of the bullet's epicyclic motion, given in Table 1, Coning Motion, were calculated from BRL data in accordance with the standard formulation. However, I took the liberty of converting the rates of angular motions from the canonical units so beloved by ballisticians, radians per caliber of travel, into actual time rates in cycles per second, or hertz.

Notice that the bullet's time rate of fast-mode "oscillation" is *practically constant* at about 290 hertz throughout the flight in BRL's formulation [See Table 1.]. Per Dr. Mann, this "oscillation rate" is typically between three and seven times (4.6 times here) faster than the bullet's initial (highest) "gyration rate," or coning rate, just after launch. Oscillation is a fast-mode nodding or bobbling motion of the bullet's spin axis that resembles gyroscopic nutation and is normally due to launch disturbances caused by:

1) A worn, distorted or dinged muzzle crown,
2) A damaged or un-square bullet base,
3) Imperfect bullet obturation,
4) A crosswind at the muzzle,
5) In-bore yaw, or by
6) A static or dynamic imbalance of the bullet itself.

The initial angular size of the bullet's slow-mode "coning motion" (i.e., its cone angle **a**) begins right after launch with *whatever size oscillation motion the bullet might have been given initially*. The fast-mode oscillation amplitude subsequently damps down to an imperceptible level fairly rapidly (with this bullet, as with probably all match bullets) within the first few thousands of calibers of forward travel (i.e., within the first couple of slow-mode coning cycles). This damping out of the fast-mode oscillation of the bullet is probably what some benchrest competitors refer to as the bullets "going to sleep." The first oscillation and coning cycles start *180 degrees out of phase* with each other so that they sum to *zero* at the instant of bullet launch from the muzzle. Thereafter, the *first maximum yaw angle* occurs at just over halfway through the initial fast-mode cycle, and the axis of the bullet returns nearly to its launch direction at just more than the first full fast-mode cycle (and somewhat less nearly into its initial alignment after each subsequently more-damped fast cycle).

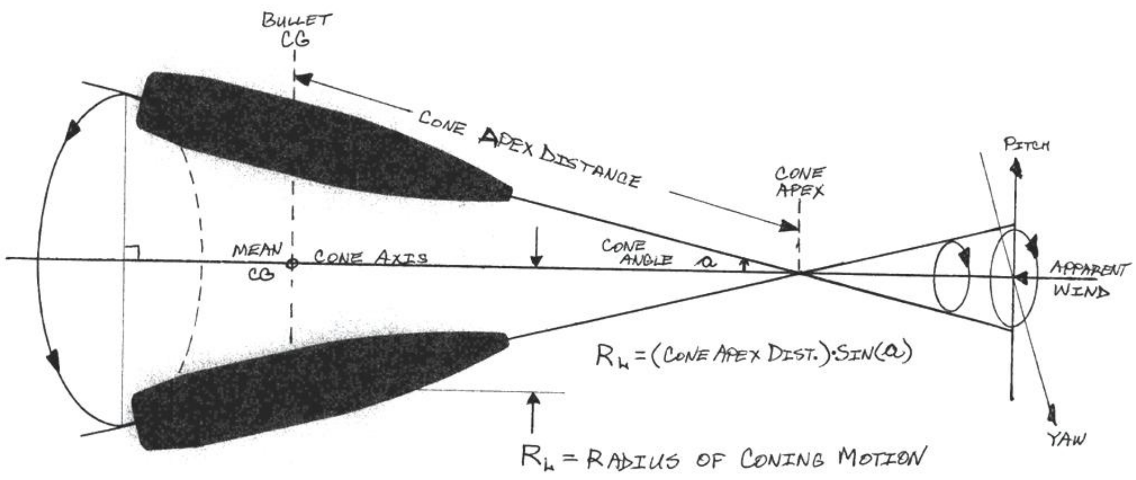

FIGURE 7. BULLET CONING MOTION

As the fast-mode motion damps out, it seems that about half of its initial oscillation amplitude adds into the slow-mode cone angle. With our example bullet here, as is probably typical of any modern bullet, the cone angle at 50 yards downrange is approximately 150 percent of its initial value, seemingly due to a contribution from the (originally equal sized) angular amplitude of the oscillation motion. As the fast-mode motion damps out, the oscillating motion of the bullet's axis seems to collapse *generally outward* toward the outer (larger radius) edges of the combined coning and oscillating motions. Then, after the oscillating motion has damped out, the cone angle at 50 yards seems to have stabilized at an average value about halfway between the initial cone angle and the bullet's first maximum yaw angle, which was *almost twice* its initial coning angle.

The Excessive Lift Problem

Recalling the example from our December 2007 *Precision Shooting* article, we launched a similar bullet with no in-bore yaw or tip-off rate, but into a uniform, horizontal, left-to-right 10 MPH crosswind. The resulting 20 MOA *angle of attack* of our spin-stabilized bullet encountering this 10 MPH crosswind at a muzzle velocity of 2600 FPS will generate a "cross-track" supersonic *lift* force in a rightward direction, and consequently, will produce a *greater* rightward wind drift effect. This rightward aerodynamic lift force is *over and above* the calculated 6.40 thousandths of a pound rightward component of the relatively large 1.135-pound average drag force over 200 yards. Earlier, we had calculated the potential aerodynamic drag force (**DYN**) to be 4.09 pounds for our example 30 caliber bullet moving at its launch speed of 2600 FPS through the old US Army METRO standard atmosphere that we used in that article. At this speed of Mach 2.32, our example bullet type has been reported to have a *coefficient of lift* **CL(a)** of **2.75**. If we multiply together this 4.09-pound potential dynamic force, times the coefficient of lift value of 2.75, and times the sine of the 20 MOA angle of attack (due to our 10 MPH crosswind), we find from this BRL formulation that the bullet must be generating an initial lift force of **0.0634 pounds** downwind, toward the right, starting as soon as the bullet clears the muzzle blast cloud. ***But, wait a minute!*** As small as this aerodynamic lift force might seem, it will accelerate our 168 grain bullet at 2.64 times the acceleration of gravity (or 2.64 g's), and it is **an order of magnitude *(ten times)* larger** than the **0.00640-pound** cross-track component of the drag force, and at least ***two*** orders of magnitude (a hundred times) larger than the assumedly negligible force that one would have expected. As we showed in the December 2007 article, the 6.4 thousandths of a pound cross-track component of the bullet's drag force is **sufficient** to account for the example bullet's **observed 1.6 MOA wind drift** over the first 200 yards of its flight.

While this presentation of the excessive lift problem is couched in terms of wind drift, any flight disturbance that causes the bullet to yaw, or to fly with a non-zero angle of attack, will cause the bullet to suffer this lift problem. We will explain in *Part II* of this article why this serious "excessive lift problem" does ***not*** cause us to experience vastly greater wind sensitivities during our outdoor matches. But we cannot do so within the bounds of the "standard formulation." Repeating for emphasis, before everyone starts blaming me for any poorer than expected groups fired in their last light-wind "trigger pulling contest," I am ***not*** saying that this hypersensitivity to crosswinds ever actually occurs, just that it would be *predicted* in the standard analytical formulation. One might suppose that this situation has persisted for so long because, as mentioned earlier, the standard analytical formulation has ***never been expected to handle real ambient winds***.

End of Part I