Barrel Stiffness Calculation

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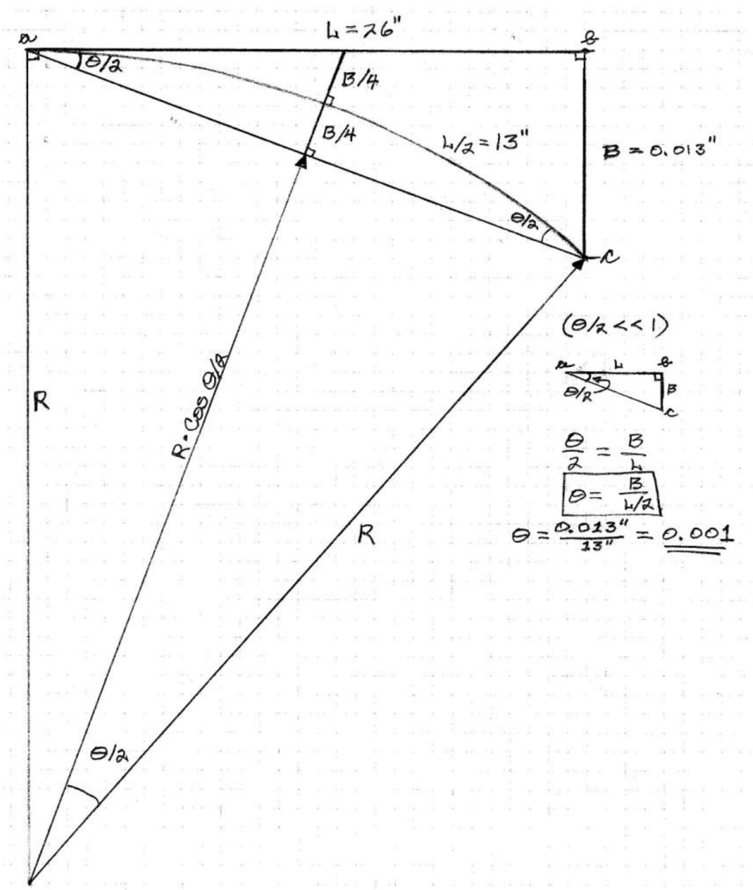
Introduction

So that we can better make design decisions, we need to quantify the stiffness of a target rifle barrel. Then we can make more informed decisions in allocating the rifle's weight budget within the rules governing a particular type of target shooting. Many small-bore target rifle barrels are made with uniform outside diameters and with barrel lengths between about 20 inches and about 36 inches. We will first develop an expression for the stiffness of a uniform diameter, fairly long and slender, solid steel rod of circular cross-section. Then we will modify that expression to apply to the thick-walled hollow cylinder of our rifle barrel. We will not consider non-cylinder barrel profiles here.

Development

Let the barrel be represented as a solid horizontal rod of uniform circular cross-section and made of a weightless material. The diameter of our barrel is **D** (in inches). The breach end of the barrel is rigidly clamped into a stout support. A length **L** (in inches) of the barrel is cantilevered out horizontally from the supporting rigid clamp. Our barrel is supporting a weight **W** (in pounds) at its distal end. While our barrel itself is weightless, it has the strength and elasticity a typical barrel steel. Young's modulus of elasticity (**E**) for this mythical steel is 30,000,000 pounds per square inch (psi). Our barrel is straight both before and after supporting the weight **W**. While it is supporting the weight **W** at its end, our cantilever barrel bends and sags by a small vertical distance **B** (in inches) at its muzzle end. The bending sag **B** is only a *few thousandths of an inch*, and our barrel is certainly well within its elastic limits.



Let us further assume that the shape of our loaded barrel is a short arc of length **L** of a very large circle of radius **R**. For example, if our barrel is 26 inches in length **L** and the bending **B** is 0.013 inch, the central angle **Θ** subtended by the barrel length **L** would be 1.0 milliradian (or one mil of mil-dot fame). The enclosed figure shows the simple geometric reasons supporting this claim. But, if 26 inches subtends one mil when seen from a distance **R**, then **R** must be 26,000 inches (or just over 0.4 miles). While this "arc of a large circle" assumption is not quite correct, it serves for our purposes here.

In the neutral plane of our loaded cantilever beam, the length **L** subtends the central angle **Θ** when seen from a distance **R** (as just explained), so that:

(#1) $$L = R*\Theta.$$

Then, at the tension-loaded upper edge of our beam, the elongated length (**L + s**) is given by:

(#2) $$L + s = (R + r)*\Theta, \quad \text{(where } r = D/2\text{)},$$

and along the bottom, compression-loaded edge, the compressed length (**L – s**) is:

(#3) $$L - s = (R - r)*\Theta.$$

So, clearly the amount of extension or compression **s** is given by:

(#4) $$s = r*\Theta.$$

Referring to the figure again, the bending distance B can be written as:

(#5) $$B/4 = R - R*Cos(\Theta/2) = R*[1 - Cos(\Theta/2)].$$

But the trigonometric function **Cos(Θ/2)** can be expanded to the series:

$$Cos(\Theta/2) = 1 - (\Theta/2)^2/2 + (\Theta/2)^4/24 - (\Theta/2)^6/720 + \ldots.$$

When **Θ/2** is a very small value in radians, we can safely ignore the higher order terms and write:

$$Cos(\Theta/2) = 1 - (\Theta/2)^2/2,$$

So that Equation **#5** becomes simply

$$B/4 = R*(\Theta/2)^2/2,$$ or multiplying through by 4:

(#6) $$B = R*\Theta^2/2.$$

Substituting for **R** from Equation **#1**:

$$B = L*\Theta/2,$$

and substituting for **Θ** from Equation **#4**:

(#7) $$B = L*s/(2r) = s*L/D.$$

Now, the standard expression for the stresses in the "extreme fibers" of a loaded beam is:

$$f = M*c/I$$

where

f = Compressive or tensile stress in an "extreme fiber" in pounds per square inch (psi)

M = Bending Moment = **L*W** (in inch-pounds)

c = Distance from neutral plane to "extreme fiber" = **D/2**, and

I = Second Moment of Cross-sectional Area of beam

$$\mathbf{I = {}_0\int^r (r^2)*(\pi r)dr}$$

$$\mathbf{= (\pi/4)*r^4}$$

$$= \mathbf{(\pi/64)*D^4} \text{ for a circular beam.}$$

Substituting these values into this standard expression, we have:

(#8) $$\mathbf{f = [L*W*D/2]/[(\pi/64)*D^4] = (32/\pi)*L*W/D^3.}$$

From the definition of Young's Modulus of Elasticity (**E**),

E = f/(s/L) = 30,000,000 psi for our assumed barrel steel,

we have

$$\mathbf{s = f*L/E,}$$

where all symbols are as previously defined.

From Equation #8, we can write this as:

$$\mathbf{s = (L/E)*[(32/\pi)*L*W/D^3] = (32/\pi)*L^2*W/(D^3*E).}$$

And Equation #7 becomes:

(#9) $$\mathbf{B = W*(32/\pi)*L^3/(D^4*E).}$$

Now, let us define a "stiffness constant" **K** for our cantilever barrel such that:

B = W/K,

then,

$\mathbf{K = (E*D^4)/[(32/\pi)*L^3],}$ (in pounds per inch of deflection).

Now, we can write the stiffness expression for a cylindrical rifle barrel of outside diameter **D** with a hole of diameter **d** bored so that its axis lies in the neutral plane simply by replacing the value $\mathbf{D^4}$ with the expression $\mathbf{(D^4 - d^4)}$, so that:

$$\mathbf{K = [E*(D^4 - d^4)]/[(32/\pi)*L^3]}$$

$\mathbf{= (2{,}945{,}000)*(D^4 - d^4)/L^3}$ **pounds/inch**, (with all dimensions are in inches).

So, the stiffness of a rifle barrel increases directly with the fourth power of its diameter, and is inversely proportional to the cube of its length.